SPECTRAL SHIFTS OF ULTRASONIC PROPAGATION THROUGH MEDIA WITH NONLINEAR DISPERSIVE ATTENUATION

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The relationship between center frequency downshift and transmitted bandwidth was investigated for a pulse with Gaussian amplitude spectrum propagating through a lossy medium with power law frequency dependence of attenuation. Tissue equivalent material was characterized by multiple narrowband attenuation measurements, via a substitution method. Power law curves were fitted to the data. The parameters of the curves were used to predict the behavior of the center frequency downshift vs. transmitted bandwidth obtained from a second experiment. The results confirm the mathematical model.

Key words: Attenuation; nonlinear attenuation; spectral shift; tissue equivalent material

I. INTRODUCTION

It is well known that ultrasound propagation in liquids and in tissues is frequency dependent. The attenuation experienced at high frequencies is generally larger than that at low frequencies. A wideband ultrasonic pulse which propagates through such a medium will suffer distortion due to the high rate of attenuation of its high frequency components. The net result is a center frequency downshift of its power spectrum. For materials with a linear frequency dependence, this shift is proportional to the integrated attenuation experienced by the pulse and to the squared bandwidth of the pulse [1,2]. In recent years it has been suggested in the literature [3 - 5] that the frequency dependence of tissue attenuation may be approximated by a power law relationship, with the exponent assuming values between 1 and 2. It is indeed this exponent which might contain information about tissue state.

Dines and Kak [1] have demonstrated the validity of modeling ultrasonic power spectra in pulsed applications by Gaussian functions suggested earlier by Serabian [6], and have derived equations for the frequency shift as a function of integrated attenuation for linear attenuation dispersion. Merkulova [7] has calculated the frequency shift of a Gaussian pulse for even powers of the frequency dependence of attenuation.

In this work we have derived the general case of frequency shift of a Gaussian pulse in a nonlinear power law attenuating medium of arbitrary exponent. The resulting equation lends itself to a numerical solution on a computer.

Two experiments were performed on tissue equivalent material in order to confirm the theory. The first experiment consisted of multiple narrowband attenuation measurements in the target material at a number of frequencies, using a substitution technique [8]. A power law curve of the
form \[ \alpha_0 f^n \] was fitted to the data, where \( \alpha_0 \) and \( n \) are the parameters of the curve, and \( f \) is frequency. Using the parameters obtained in this way, the equation developed in the paper (Eq. (5)) was used to predict the center frequency shift as a function of transmitted bandwidths. The theoretical prediction based on the first experiment was then compared with the experimental results of the second experiment.

In conclusion, this work derives and verifies the equation relating the center frequency downshift as a function of bandwidth to the target material power law attenuation parameters \( \alpha_0 \) and \( n \). As a corollary, the results indicate that the Gaussian spectral model is a reasonable one for this type of measurement, confirming the conclusions of Dines and Kak [1].

II. THEORY

It is assumed that a reference ultrasonic pulse propagating through a lossless medium has a time domain Gaussian envelope. The Fourier amplitude spectrum of the pulse, therefore, also has a Gaussian envelope [9] of the form

\[
|T(f)| = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(f-f_0)^2}{2\sigma^2} \right],
\]  

(1)

where \( |T(f)| \) = Fourier amplitude spectrum of the reference pulse in a lossless medium

\( \sigma^2 = \) variance of the spectrum

\( f_0 = \) center frequency of the spectrum.

It is further assumed that the attenuation of sound intensity in the medium of propagation is in general nonlinearly dispersive with frequency, and is of the exponential form:

\[
|H(f)| = \exp \left[ -2\alpha_0 f^n Z \right],
\]  

(2)

where \( |H(f)| \) = frequency dependent attenuation filter function

\( n = \) exponent of frequency dependence (typically \( 1 \leq n \leq 2 \) for tissue)

\( Z = \) total pulse propagation distance

\( \alpha_0 = \) amplitude attenuation frequency dependence coefficient of the medium (in Nepers cm\(^{-1}\) MHz\(^{-n}\)).

In a lossy medium, the received spectrum, \( |R(f)| \) is therefore given by:

\[
|R(f)| = |T(f)| |H(f)| = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(f-f_0)^2}{2\sigma^2} + 2\alpha_0 f^n Z \right].
\]  

(3)

After traversing a path in the medium, the pulse \( |R(f)| \) has experienced a downshift in its center frequency. The center frequency \( f_c \) is defined as the frequency at which the amplitude spectrum slope is zero. In order to find the downshifted center frequency, we differentiate \( |R(f)| \) with respect to \( f \) and set to zero:

\[
\frac{d}{df} |R(f)|_{f=f_c} = \left\{ \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(f-f_0)^2}{2\sigma^2} + 2\alpha_0 f^n Z \right] \right\} \left\{ -\frac{(f-f_0)}{\sigma^2} -2\alpha_0 Z f^{n-1} \right\} = 0.
\]  

(4)

283
The first term in the equation is $\neq 0$ for all finite frequencies. We therefore set the second term to zero. The resultant equation becomes

$$2\alpha o Z o^2 f_c^{n-1} + f_c - f_o = 0.$$ (5)

This equation describes the location of the downshifted center frequency $f_c$ as a function of $f_o$, $o$, $\alpha$, Z, and n. The equation cannot be solved in closed form, with the exception of the simple cases of interest (see Discussion) when $n = 1$ or 2. These cases, however, can be solved in order to show the general trends. Other cases can be solved by using an iterative procedure on a computer. The case of $n = 3$ has a closed form solution but is of no practical interest.

1. Linear Case ($n = 1$)

In this case, Eq. (5) reduces to [1,2]

$$f_c = f_o - 2\alpha o Z o^2.$$ (6)

Note that the downshifted center frequency $f_c$ is equal to the transmitted center frequency, $f_o$, less a difference frequency $2\alpha o Z o^2$. Thus the degree of center frequency downshift is a linear function of the product of the attenuation constant $\alpha$ (property of the medium), the distance Z (an experimental variable), and the variance of the spectrum of the transmitted pulse (an experimental variable). (See Appendix for the relationship between variance and bandwidth). It is also worthy to note that CW signals ($o^2 = 0$) will exhibit no center frequency shift.

2. Quadratic Case ($n = 2$)

Equation (5) becomes

$$f_c = \frac{f_o}{4\alpha o Z o^2 + 1}.$$ (7)

We again see that the downshift depends on the now dimensionless term $\alpha o Z o^2$, but in a nonlinear fashion.

III. MATERIALS AND METHODS

Two experimental procedures are used to verify the theory. The first procedure involved the determination of the frequency dependence on the attenuation coefficient in tissue equivalent material (TEM) at constant room temperature ($22^\circ C \pm 1^\circ C$). The second procedure involved the measurements of the spectral shifts experienced by pulses of variable bandwidth propagating through this medium. The results of the first experimental procedure were then used to predict the results of the second experiment via the theory.

1. Experimental determination of the frequency dependence of attenuation.

A multifrequency, narrowband, pulse echo substitution method was used to determine the frequency dependence on the attenuation in the target materials. The experimental setup is shown in figure 1. Long ($>10$ $\mu$s) gated sinewave bursts were chosen at 0.5 MHz increments from 1.5 to 5 MHz. The transducer was placed at the top of a degassed water filled glass.

1 Courtesy of Acoustic Standards Corp., Houston, TX 77035

284
Fig. 1 Experimental setup. The blocks within the dotted lines are added for the frequency shift experiment.

cylinder partially submerged in a constant temperature bath. The bottom of the cylinder was located near the focal region of the transducer, and contained a thick (2.5 cm) acrylic plane reflector aligned for normal wave incidence. Reference reading of peak echo amplitudes were recorded for each of the frequencies above.

The target material was then inserted between the transducer and the reflector. The attenuation which had to be removed from the circuit in order to match the concomitant reference measurements for each frequency was considered the total bulk attenuation of the target material. The attenuation coefficient at each frequency was computed as

$$\alpha = \frac{\alpha_z}{Z},$$

where $\alpha$ = the attenuation coefficient at a specific frequency in Nepers/cm

$\alpha_z$ = the total bulk attenuation of the target material in Nepers.

A power law fit to the data was performed using a HP-85 computer and standard library programs. The fit was of the form

$$\alpha_o f^n,$$

where

$\alpha_o$ = the attenuation frequency dependence coefficient, in Nepers cm$^{-1}$ MHz$^{-n}$

$^2$Hewlett Packard Corp., Palo Alto, CA
Fig. 2. Typical spectrum analyzer outputs. Horizontal scale is 500 kHz/ div.; vertical scale is 10 dB/div. Note 5 MHz pip marker added to spectrum.

a. Wideband transmitted spectrum in a lossless medium.
b. The spectrum in (a) after travel in a lossy medium. 
   Observe large downshift in center frequency (≈ 600 kHz).
c. Narrower band transmitted spectrum in a lossless medium.
d. The spectrum in (c) after the same travel through the lossy medium. Observe a smaller center frequency downshift (≈ 300 kHz).
e. Narrow band transmitted spectrum in a lossless medium.
f. The spectrum in (e) after the same travel through the lossy medium. Observe the small degree of center frequency shift (≈ 50 kHz).

f = frequency, in MHz
n = exponent (usually 1 ≤ n ≤ 2).

2. Experimental determination of spectral shift vs. bandwidth.

These measurements were performed using the setup described previously, with the addition of a Tektronix 7L12 spectrum analyzer and a time domain RF gate (Fig. 1). The transducer used was a special wideband transducer\(^3\). Gated sinewaves of various durations (and thus bandwidths) and of

\(^3\)Courtesy of Philips Ultrasound, Inc., Santa Ana, CA
Table 1. Results of narrowband attenuation measurements in tissue equivalent material.

<table>
<thead>
<tr>
<th>Frequency, MHz</th>
<th>Tissue Equivalent Material Attenuation, Nepers/cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.08</td>
</tr>
<tr>
<td>2.0</td>
<td>no data available</td>
</tr>
<tr>
<td>2.5</td>
<td>0.12</td>
</tr>
<tr>
<td>3.0</td>
<td>0.16</td>
</tr>
<tr>
<td>3.5</td>
<td>0.19</td>
</tr>
<tr>
<td>4.0</td>
<td>0.22</td>
</tr>
<tr>
<td>4.5</td>
<td>0.26</td>
</tr>
<tr>
<td>5.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

approximately 4.8 MHz center frequency were applied to the transducer (See Appendix). The spectra of the echoes from the acrylic block were recorded in the presence of water (assumed to be non-attenuating) or an attenuating target. The shift in the center frequency of the pulse was determined from the spectrum analyzer display. (See Fig. 2a-d.)

IV. RESULTS

The results of the experimental procedure for determining the frequency dependence of the attenuation in TEM are shown in table 1. The best (coefficient of determination = 0.99) power law fit to the data was calculated to be

\[ \alpha(f) = 0.0329f^{1.38} \text{ Nepers cm}^{-1} \]  

(10)

These parameters were used to numerically solve Eq. (5) on the computer. The solutions are shown in figure 3 as the solid curve relating the shift in the frequency to the bandwidth employed. The experimental data points of the shift in center frequency vs. bandwidth are shown.

V. DISCUSSION

The experimental results suggest that the mathematical model for the shift in center frequency as a function of transmitted bandwidth is reasonably accurate, even though the model assumes a Gaussian transmitted spectrum, while the actual spectrum was approximately of a sinc x form. The Gaussian form was used as a mathematical convenience; other spectra could have been used. The conclusion reached by Dines and Kak [1] is thus corroborated.

The analysis, while different than Merkulova's [7], yields similar results. Furthermore, Merkulova's closed form solutions are only valid for n = even integer. The analysis given here gives closed form solutions for n = 1,2, which are two special cases of interest. (Some biological tissues are thought to behave linearly with frequency, while liquids behave classically with square law dependence [1]) However, the analysis for 1 ≤ n ≤ 2 is also of importance, particularly for tissue characterization. The numerical solution of Eq. (5) by the computer allows the characterization of materials with an arbitrary value of n, and in particular those with 1 ≤ n ≤ 2.

Errors in the experimental measurements are difficult to assess because the judgment of center frequency had to rely on visual inter-
Fig. 3 Frequency downshift vs. transmitted bandwidth in tissue equivalent material. The solid curves are theoretical prediction from Eq. (5). The experimental points are results from the frequency shift experiments. The total propagation distance is 10.16 cm.

Interpretation of the spectrum analyzer display. For wideband pulses, which contain most of the information about the material, it is even more difficult to determine the true center frequency due to the flatness of the spectrum. Small shifts (up to 75 kHz) in the transmitted center frequency as a function of bandwidth were noted at the output of the burst generator. The discrepancy in the two sets of data, on the order of 25-75 kHz (see Fig. 3) is most likely due to target temperature variations (±1°C) which affect both $\alpha_0$ and $n$.

VI. APPENDIX: Relationship between the Variance and Half Amplitude Bandwidth of Gaussian Spectra

The half amplitude bandwidth can be found by setting the Gaussian spectrum envelope to half its maximum value, i.e.

$$\exp\left[-\frac{(f-f_0)^2}{2\sigma^2}\right] = 1/2. \quad (11)$$

Solving for $f_{h,l}$, the high and low frequencies for which the amplitude is 1/2, we get

$$f_{h,l} = f_0 \pm 1.18\sigma, \quad (12)$$

where $f_0$ is the center frequency and $2.36\sigma$ is the half amplitude bandwidth.
REFERENCES


