**MOIRÉ UNDERSAMPLING ARTIFACTS IN DIGITAL ULTRASOUND IMAGES**

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In modern ultrasonic digital sector scanners, the interaction between angulated line-of-sight vectors and a square pixel matrix results in Moiré artifacts in the image. These artifacts are a result of non-sampling of some pixels under certain conditions. The conditions which govern the appearance and progression of these artifacts are discussed, and an actual hardware demonstration is given.

Key words: Digital ultrasound; Moiré artifacts; sector scanner; undersampling.

I. INTRODUCTION

In recent years, many significant advances have occurred in the field of diagnostic ultrasound. Among these are the advent of the automated sector scanner [1], and the wide acceptance of the digital scan converter as a replacement for the older analog machine [2].

The use of the automated sector scanner in conjunction with a digital scan converter often results in artifactual Moiré patterns in the image [3,4]. These artifacts are manifestations of the interaction between the sector geometry and the pixeled nature of the image, and are dependent on pixel matrix size, transducer angular velocity, ultrasound pulse repetition rate, transducer location with respect to the pixel matrix, and any symmetry shift of the transducer angulations from a given pixel axis.

The artifacts appear as blank unsampled pixels arranged in linear and nonlinear patterns. Ideally, the patterns are formed by pixels which are connected to each other, often in the corners and sometimes along the edges. Due to quantization errors, however, the observed patterns are often disjointed.

It is shown that the condition for adequate sampling (and thus the absence of artifacts) is that

$$\omega T < \tan^{-1} \left[ \frac{1}{(N-1)} \right],$$

where \( \omega \) is the angular velocity of the transducer, \( T \) is the time interval between two ultrasonic transmit pulses, and \( N \) is the number of pixels along one side of the square \( N \times N \) pixel matrix. The underlying assumption is that the ultrasonic lines-of-sight are straight (i.e., no transducer motion...
Fig. 1 An adequately sampled 32 x 32 pixel matrix. The angle between
type[s] is $\theta = \tan^{-1}(1/N) = \tan^{-1}(1/32) = 1.791^\circ$.


takes place between the start and finish of a given line-of-sight). When
$\omega$ and/or $T$ is increased such that

$$\omega T > \tan^{-1}[1/(N-1)],$$

unsampled pixels appear in the image. The artifacts first appear in
locations far from the transducer. As the product $\omega T$ increases further,
unsampled pixels appear progressively closer to the transducer. In the
limit when $\omega T = \pi/2 >> \tan^{-1}[1/(N-1)]$, the whole image is filled with
unsampled pixels.

Similar artifacts can also occur in digital ultrasound B-scan equip-
ment. However, due to the quasi-random motion and angulation of the hand-
held transducer in such machines, meaningful analysis of such artifacts is
difficult. In addition, the often overlapping scanning motion tends to
obscure and hence minimize these artifacts. On the other hand, automated
sector scanners tend to produce a marked artifact, due to the programmed
motion of the transducer.

II. ANALYSIS

For the purpose of this analysis, we assume an N x N square pixel
matrix (Fig. 1). The boundaries between pixels along the horizontal axis
are labeled $X_1, X_2, \ldots, X_N$, and the boundaries between pixels along
the vertical axis are labeled $Y_1, Y_2, \ldots, Y_N$. We further assume that
the transducer is located at the top left corner of the matrix and that
its angular velocity is uniform. The equal angles $\theta$ between radial vectors
is given by $\theta = \omega T$. In this figure, we observe that a special case was
chosen such that $\theta = \tan^{-1}(1/N)$. Inspection of all pixels in the matrix
reveals that at least one vector passes through each and every pixel, such
that all pixels are adequately sampled, without leaving any gaps. By in-
spection, in all cases where \( \Theta < \tan^{-1}(1/N) \), all pixels are adequately
sampled in a similar manner. If we observe the effect of increasing \( \Theta \) on
the bottom left pixel in the matrix, we conclude that when \( \tan^{-1}1/N \leq \Theta <
\tan^{-1}1/(N-1) \), this pixel is still adequately sampled. However, when
\( \Theta = \tan^{-1}1/(N-1) \), the vector passes only through the corner of the pixel
(Fig. 2), and thus this pixel becomes the first one to be unsampled.

When either the angular velocity and/or the time interval between
transducer firings increase such that \( \Theta > \tan^{-1}[1/(N-1)] \), artifacts
begin to appear in the image. This is shown in figure 3. The figure
shows the special case where the angle \( \Theta \) has been increased somewhat to
\( \Theta = \tan^{-1}(5/4N) \), corresponding (near the vertical axis) to 5/4 pixels in
the X direction per N (full scale) pixels in the Y direction (for the
case where \( N = 32 \), \( \tan^{-1}[1/(N-1)] = 0.14 < \tan^{-1}[5/4N] = 0.15 \). We
wish to find \( Y_{trc} \), the Y coordinates of the top right corners of the
artifactual (shaded) gaps, marked by bold dots. We define an ensemble as
a collection of unsampled gaps whose corners or edges are connected. En-
sembles are ranked \( k \), where \( k = 1, 2, 3 \ldots \) etc., and the ultrasonic ray
vectors are numbered \( n \), where \( n = 1, 2, 3 \ldots \) etc. For ensemble \( k = 1 \),
we have \( X_1 = Y_{trc} \cdot \tan \Theta \), and therefore: \( Y_{trc} = X_1/\tan \Theta = 1/\tan \Theta = 4N/5 \). This is the Y coordinate of the top right corner of ensemble \( k = 1 \). En-
semble \( k = 1 \) is seen in the figure (and in all subsequent figures) to
consist of only one long gap.

Ensemble \( k = 2 \) consists of a family of corner-connected gaps and the
\( Y_{trc} \) values are given (for \( n > 4 \)) by

\[
\begin{align*}
X_5 &= Y_{trc} \tan 4\Theta \\
X_6 &= Y_{trc} \tan 5\Theta \\
&\vdots \\
X_{n+1} &= Y_{trc} \tan (n\Theta).
\end{align*}
\]

Ensemble \( k = 3 \) similarly has \( Y_{trc} \) points such that (for \( n \geq 8 \))

\[
\begin{align*}
X_{10} &= Y_{trc} \tan 8\Theta \\
X_{11} &= Y_{trc} \tan 9\Theta \\
&\vdots \\
X_{n+2} &= Y_{trc} \tan (n\Theta).
\end{align*}
\]

Ensemble \( k = 4 \) has \( Y_{trc} \) points such that (for \( n \geq 11 \))

\[
\begin{align*}
X_{14} &= Y_{trc} \tan (11\Theta) \\
X_{15} &= Y_{trc} \tan (12\Theta) \\
&\vdots \\
X_{n+3} &= Y_{trc} \tan (n\Theta).
\end{align*}
\]

Ensemble \( k = 5 \) has \( Y_{trc} \) points such that (for \( n \geq 13 \))

\[
\begin{align*}
X_{17} &= Y_{trc} \tan (13\Theta) \\
X_{18} &= Y_{trc} \tan (14\Theta) \\
&\vdots \\
X_{n+4} &= Y_{trc} \tan (n\Theta).
\end{align*}
\]
Fig. 2 A marginally sampled matrix. The angle $\theta = \tan^{-1}(1/N - 1) = \tan^{-1}(\frac{1}{37}) = 1.848^\circ$. Note that only the left bottom pixel is unsampled, forming ensemble #1. The bold dot is the right top corner of the gap.

Fig. 3 A lightly unsampled matrix. The angle $\theta = \tan^{-1}(\frac{5/4}{32}) = 2.236^\circ$. Note patterns (ensembles) beginning to form far from the transducer. Bold dots are the top right corners of the unsampled gaps. Only unsampled whole pixels are shaded.
Fig. 4 A lightly unsampled matrix. \( \Theta = \tan^{-1}\left(\frac{4/3}{32}\right) = 2.384^\circ \).

In general, the pattern for all ensembles \( k \) is therefore

\[
Y_{\text{trc}} = \frac{X_{n+k-1}}{\tan (n\Theta)}.
\]  (3)

Figures 4, 5 and 6 show undersampling artifacts for cases where 
\( \phi = \tan^{-1} 4/3N \), \( \phi = \tan^{-1} 3/2N \) and \( \phi = \tan^{-1} 2/N \), respectively. For all cases it can be demonstrated, as was done above for the case of \( \phi = \tan^{-1} 5/4N \), that Eq. (1) holds.

Case of Symmetry Shift

In all previous cases, the scanning pattern was assumed to be evenly symmetric about the ordinate. Figure 6 showed the case of a scan with 
\( \phi = \tan^{-1} (2/N) \) with even symmetry. Figure 7 shows what happens when a symmetry shift angle \( \Theta' \) is introduced. In this case \( \Theta' = \tan^{-1} (1/N) \).

For this case, ensemble \( k = 1 \) is at

\[
Y_{\text{trc}} = \frac{X_1}{\tan \Theta} = \frac{1}{\tan \Theta'} = N;
\]

thus this ensemble does not exist within the square matrix.

For \( k = 2 \)

\[
X_2 = Y_{\text{trc}} \tan (\Theta + \Theta')
\]

\[
X_3 = Y_{\text{trc}} \tan (2\Theta + \Theta')
\]

\[
\vdots
\]

\[
X_{n+1} = Y_{\text{trc}} \tan (n\Theta + \Theta')
\]

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Fig. 5 A moderately unsampled matrix. \( \theta = \tan^{-1}\left(\frac{3/2}{32}\right) = 2.686^\circ \).

Fig. 6 A heavily unsampled matrix. \( \theta = \tan^{-1}\left(\frac{2}{32}\right) = 3.576^\circ \).
Fig. 7 Effect of asymmetry of $\Theta' = \tan^{-1}(\frac{1}{32}) = 1.790^\circ$, on figure 6. Note shifting of ensemble #1 and all other ensembles.

Similarly, for $k = 3$

$$X_k = Y_{\text{trc}} \tan (2\Theta + \Theta')$$

$$X_{k+2} = Y_{\text{trc}} (n\Theta + \Theta');$$

thus in this case ensembles obey the equation

$$Y_{\text{trc}} = \frac{X_{n+k-1}}{\tan (n\Theta + \Theta')}$$

This equation reduces to Eq. (1) in the symmetric case ($\Theta' = 0$).

III. DISCUSSION AND CONCLUSIONS

In all cases demonstrated, a 32 x 32 square pixel grid was used for illustration only. The demonstration is independent of this particular grid size and holds for any square grid size $N \times N$, since the angles $\Theta$ and $\Theta'$ were defined as the arctangents of the ratio of number of square pixels and not as absolute angles.

The threshold for the occurrence of artifacts is $\Theta = \tan^{-1} [1/(N-1)]$. This value of $\Theta$ is the largest for which each pixel is sampled. If the interval between ultrasonic pulses decreases and/or the angular velocity decreases such that $\Theta < \tan^{-1} [1/(N-1)]$, the image is adequately sampled or oversampled and no gaps appear in the image. On the other hand, if the interval and/or the angular velocity increases such that $\Theta > \tan^{-1} [1/(N-1)]$, artifacts will occur. This also means that increasing the physical size of the pixel matrix for a given $\omega$ and $T$ will increase the degree of under-
Fig. 8 Actual digital simulation of undersampling artifacts in a 32 x 32 pixel matrix. The white areas are adequately sampled. The black gaps are artifactual. These figures should be compared to the respective diagrams: a) $\Theta = \tan^{-1}(1/32) = 1.791^\circ$. b) $\Theta = \tan^{-1}(\frac{32}{\sqrt{3}}) = 2.236^\circ$. c) $\Theta = \tan^{-1}(\frac{32}{2}) = 2.384^\circ$. d) $\Theta = \tan^{-1}(\frac{272}{32}) = 2.686^\circ$. e) $\Theta = \tan^{-1}(\frac{272}{32}) = 3.576^\circ$. Note also that ensembles are disjointed in many cases where digital truncation has occurred. The simulation is approximate due to uncertainties in the positions of the vectors.
sampling. This will cause artifacts in a critically sampled image where \( \Theta = \tan^{-1} \left[1/(N-1)\right] \), and worsen them in an already undersampled image.

The general equation for the \( Y \) axis locations of the top right corners of the undersampled areas was found to be

\[
Y_{trc} = \frac{X_{n+k-1}}{\tan (n\Theta + \Theta')} ,
\]

where \( Y_{trc} \) is the (analog) locations of the adjoining corners in the ensembles, \( \Theta' = \tan^{-1} (M/N) \), where \( M \) is the number (or fraction) of pixels tra-
versed in the X direction between the axis and the first vector, and N is the total number of pixels in the Y direction, n = the vector number and k = the ensemble number. The actual quantized Y location of an unsampled pixel corner may generally be found by rounding off the Y location to the nearest pixel boundary or by digital truncation. This means that actual scans will not show the ensembles in the precise way they were depicted here. This point is illustrated in figures 2-7 by the shaded pixels. These pixels are the whole pixels contained vertically between subsequent Y,te coordinates; partial pixels are truncated. Figures 8 (a) thru (e) show actual digital simulations of such artifact generation.

ACKNOWLEDGEMENT

This work was supported in part by Ausonics Pty Ltd., and is published with their permission. The artwork was done by Karen Ophir.

REFERENCES


