ON THE VALIDITY OF THE LINEAR APPROXIMATION IN THE PARAMETRIC MEASUREMENT OF ATTENUATION IN TISSUES

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Abstract—A known method for the determination of the attenuation coefficient of tissues involves the measurement of the center frequency downshift of Gaussian spectra. The tissue is generally assumed to have linear frequency dependent attenuation. This assumption results in a simple relationship between the spectral downshift and the attenuation coefficient. However, recent studies have shown that many tissues exhibit nonlinear frequency dependent attenuation. In this paper we investigate the consequences of applying the linear assumption to non-linear cases. These studies show that even a small deviation from linearity results in significant errors in the determination of the attenuation parameters.

Key Words: Ultrasound, Attenuation, Material parameters, Parametric methods, Gaussian spectrum, Linear and nonlinear frequency dependence, Blood, Tissue mimicking material.

INTRODUCTION
The attenuation of ultrasound passing through a lossy material is frequency dependent. This property has potential for quantitative tissue characterization (Miller et al., 1976; Kak and Dines, 1978; Lizzi and Elbaum, 1979; Goss et al., 1979; Bamber and Hill, 1981a; Bamber and Hill, 1981b; Jones et al., 1981; Kuc and Taylor, 1982; Fink and Hottier, 1982; Nicholas, 1982). In general, this frequency dependence is expressed as (Hill, 1978)

\[ \alpha(f) = \alpha_0 f^n, \]  

where \( f \) = frequency; \( \alpha(f) \) = frequency dependent attenuation coefficient; and \( \alpha_0, n \) = attenuation parameters characterizing the material. The quantitative measurement of attenuation is thus reduced to determining the parameters \( \alpha_0 \) and \( n \). It can be seen from this equation that higher frequencies are more attenuated than lower frequencies, resulting in a spectral distorsion which appears as a downshift of the center frequency of an incident wideband pulse, and in order to obtain a relationship between the spectral downshift and the attenuation parameters, it is necessary to assume a mathematical model for the incident spectrum. The common model for the spectrum is a Gaussian (Merkulova, 1967; Kuc et al., 1976; Dines and Kak, 1979). Most of the work reported in the literature concerning this parametric method, assumes that the attenuation in tissues increases linearly with frequency. This assumption simplifies the analysis significantly. However, recent studies have shown that many tissues exhibit a nonlinear frequency dependent attenuation† (Goss et al., 1979; Jones et al., 1981; Nicholas, 1982). In particular, it was observed that \( n = 1.5 \) for spleen and \( n = 2 \) for white matter (Nicholas, 1982).

Recently Ophir and Jaeger (1982) have derived a relationship between the downshifted center frequency and the material parameters for an arbitrary value of \( n \). The form of this relationship clearly indicates that assuming linear frequency dependent attenuation for tissues which are nonlinear might result in errors in the determination of the material parameters. The purpose of this work is to investigate these errors. We have shown theoretically and experimentally that even for small deviations from linearity, significant errors in the estimation of the material parameters are encountered.

ANALYSIS
The downshifted center frequency for a Gaussian pulse can be calculated by starting from the equation derived by Ophir and Jaeger (1982). This equation is given by

\[ \Delta f = f_0 - f_c = 2n\alpha_0 Z \sigma^2 f_c^{n-1} \]  

where \( \Delta f \) = center frequency downshift; \( f_0 \) = center frequency of the incident spectrum; \( f_c \) = center frequency of the downshifted spectrum; \( Z \) = total propagation distance; and \( \sigma^2 \) = variance of the spectrum.

†The term "nonlinear" used here and throughout this paper refers to the nature of the frequency dependence of attenuation, and is not to be confused with nonlinear attenuation which depends on intensity.
Equation (2) can be rewritten as

$$\Delta f = 2n\alpha_0 Z \sigma f_0 n^{-1} \left[1 - \frac{\Delta f}{f_0}\right]^{-1}.$$  (3)

In general, the quantity $\Delta f/f_0 \ll 1$. For instance, Kuc et al. (1976) determined that for a 2-cm formalin fixed liver specimen, $\Delta f = 0.058$ MHz at $f_0 = 2$ MHz. Thus $\Delta f/f_0 \approx 0.03$ and can be ignored. With this approximation, eqn (3) can be written as

$$\Delta f \approx 2n f_0^{n-1} \alpha_0 Z \sigma^2 = 2\kappa Z \sigma^2,$$  (4)

where

$$\kappa = (nf_0^{n-1}) \alpha_0.$$  (5)

Experimentally, the quantity $\Delta f$ is always measured, while $Z$ and $\sigma^2$ are known, and the quantity $\kappa$ is determined from eqn (4). This procedure is applicable to any linear or nonlinear material.

For the linear case, the value of $\kappa$ becomes

$$\kappa = \alpha_0,$$  (6)

which is a true material parameter.

For the nonlinear case, $\kappa$ is given by eqn (5), and thus is not solely determined by the material, since it also depends on the center frequency. When we measure the attenuation in a nonlinear material, but assume that the material is linear and that eqn (6) applies, we suffer an error in the determination of the true value of $\alpha_0$. We define this error as

$$\epsilon = \frac{\kappa - \alpha_0}{\alpha_0} \times 100\%.$$  (7)

Substituting for $\kappa$ from eqn (5), we get

$$\epsilon = (nf_0^{n-1} - 1) \times 100\%.$$  (8)

Note that the magnitude of the error increases with $n$ and $f_0$. The percent error was calculated as a function of $n$ for several values of $f_0$, and is shown in Fig. 1. From this figure we observe that under typical experimental conditions, the magnitude of $\alpha_0$ is always overestimated, and the magnitude of error can be significant ($>10\%$) even for values of $n$ which are only slightly greater than 1. For example, for $n = 1.1$ and $f_0 = 4$ MHz, $\epsilon$ is as large as 26%. Even for $n = 1.05$, $\epsilon$ equals 12%. When $n$ reaches the value of 1.3, $\epsilon$ approaches 100%.

It should be noted that for the linear case, the experimental verification

In order to verify the above error analysis, two experiments were performed on two different samples. The samples were non-clotted human whole blood and tissue mimicking material (TMM†). The first experiment involved the determination of the material parameters $\alpha_0$ and $n$ of the samples using classical narrowband substitution methods. The values so determined were considered the true values of the material parameters. The second experiment involved the determination of the parameter $\kappa$ from multiple measurements of the center frequency shifts experienced by pulses of variable bandwidths propagating through these media.

The sample materials were enclosed in a plexiglas® cylinder of 10 cm dia. and 5 cm length. The cylinder was closed at both ends with 0.5 mil thick mylar membranes. The attenuation caused by this membrane was found to be negligible. The samples were degassed for 6 hr under laboratory vacuum. The blood sample used was outdated and obtained from the blood bank.

A multifrequency, narrowband, pulse echo substi-
tution method was used to determine the frequency dependence of the attenuation in the target materials. The experimental setup is shown in Fig. 2. Long (≥10 μsec) gated sinewave bursts were chosen at 0.5 MHz increments from 2 to 5 MHz. The transducer was a 19-mm, circular focused wide-band transducer, which was placed at the top of a degassed water tank maintained at 22 ± 1°C. The bottom of the tank was located in the focal region of the transducer, and contained a thick (2.5 cm) acrylic plane reflector aligned for normal wave incidence. Reference readings of peak echo amplitudes were recorded for each of the frequencies above. The target material was then interposed between the transducer and the reflector. The attenuation which had to be removed from the circuit in order to match the concomitant reference measurements for each frequency was considered the total bulk attenuation of the target material. The attenuation coefficient at each frequency was computed as

\[ \alpha(f) = \frac{\alpha_r(f)}{Z}, \]  

(9)

where \( \alpha_r(f) \) = the total bulk attenuation of the target material at frequency \( f \). A fit of eqn (1) to the data was performed to determine \( \alpha_0 \) and \( n \). The data and the best fit curves are shown in Fig. 3. The values obtained for blood were

\[ \alpha_{0,\text{blood}} = 0.035 \text{ neper cm}^{-1} \text{ MHz}^{-n}, n = 1.18, \]

and the values obtained for tissue mimicking material were

\[ \alpha_{0,\text{TMM}} = 0.033 \text{ neper cm}^{-1} \text{ MHz}^{-n}, n = 1.38. \]

The frequency shift measurements were performed using the same setup, with the addition of the Tektronix 7L12 spectrum analyzer and a time domain RF gate. Gated sinewaves of various durations (and thus bandwidths) and of 4.8 MHz center frequency were applied to the transducer. The spectra of the echoes from the plane reflector were recorded on the XY recorder in the presence and absence of the samples. The center frequency downshift was taken as the shift of the peak position of the spectrum introduced by the attenuating material. The observed frequency downshift was plotted as a function of the bandwidth squared, and the parameter \( \kappa \) was obtained by fitting this data to eqn (4). Even though the theory was developed for Gaussian spectra, this analysis is valid.

![Fig. 2. Block diagram of the experimental setup used to measure the attenuation. The blocks within the dashed lines are added for the frequency shift measurements.](image-url)
for cases where the spectrum is non-Gaussian. In the experiments reported here, an approximate sinc \((x)\) function was used for convenience. It has been shown by Narayana and Ophir (1983) that this use is valid when \(\pi \Delta f / B < 1\) where \(B\) is the 6-dB bandwidth. This condition is satisfied in the present experiments. Plots for blood and tissue mimicking material are shown in Fig. 4. The bandwidth is the half amplitude bandwidth and is equal to 2.36\(\sigma\). The figure shows linear dependencies of \(\Delta f\) on \(\sigma^2\) for both blood and TMM. This behavior is consistent with eqn (4), with the slope given as \(2\kappa Z\). If we purposely ignore the nonlinear behavior, and instead assume linearity, we have \(\alpha_0 = \kappa\). The value of \(\alpha_{0,\text{blood}}\) so determined is 0.048 neper cm\(^{-1}\) MHz\(^{-1}\), and for the tissue mimicking material \(\alpha_{0,\text{TMM}}\) is 0.076 neper cm\(^{-1}\) MHz\(^{-1}\). These values of \(\alpha_0\) for blood and TMM are higher than the corresponding values of \(\alpha_0\) determined using the substitution method. Since the substitution results show that both materials are in fact nonlinear, the values determined from Fig. 4 are not \(\alpha_0\), but rather \(nf_0^{-n} \alpha_0\). Taking the \(\alpha_0\) and \(n\) values obtained from the substitution method, and an \(f_0\) of 4.8 MHz we get from eqn (5)

\[
\alpha_{0,\text{blood}} = 0.030 \text{ neper cm}^{-1} \text{ MHz}^{-n}
\]

and

\[
\alpha_{0,\text{TMM}} = 0.030 \text{ neper cm}^{-1} \text{ MHz}^{-n}.
\]

These corrected values are close to the true values measured by substitution.

**DISCUSSION**

The purpose of this paper is to demonstrate the problems which are associated with the use of linear analysis in the investigation of attenuation of unknown tissues. For all linear and nonlinear cases, the frequency shift \(\Delta f\) is related to the variance \(\sigma^2\) as

\[
\Delta f = 2\kappa Z \sigma^2.
\]

The parameter \(2\kappa Z\) is obtained experimentally as the slope of the lines in Fig. 4, from which \(\kappa\) is extracted. In the linear case, \(\kappa = \alpha_0\) and thus is a true material
parameter. If the material does in fact exhibit nonlinear behavior, \( \kappa \) becomes a function of \( z_0, n \) and \( f_0 \) (eqn 5), and thus becomes dependent on the experimental conditions. It is no longer a true material parameter.

Linear analysis has been used by several investigators in the description of the attenuation in liver tissue (Kuc and Taylor, 1982; Fink and Hottier, 1982). Evidence indicates that in the special case of normal liver, this may be a valid procedure, since the attenuation in this material appears to be nearly linear (Goss et al., 1979; Nicholas, 1982). In general, however, it cannot be assumed that tissue attenuation varies linearly with frequency. This applies to tissues other than liver, which exhibit values of \( n \) which range upwards of 2 (Nicholas, 1982), and for pathological tissues whose values of \( n \) are completely unknown. Moreover, we generally do not know what the properties of the tissue are before we make the measurement. We must therefore use the general equations (4) and (5) to compute the true material parameters, viz. \( z_0 \) and \( n \), in the absence of such a priori knowledge.

When examining the behavior of the attenuation of tissues, it might be intuitively concluded that a slight nonlinear behavior of attenuation vs frequency might not affect the determination of \( z_0 \) in a measurable way. However, our analysis shows that even for values of \( n \) which are slightly greater than 1, the errors due to the use of eqns (4) and (6) become significant, and in fact quickly become very large as \( n \) increases. This error is also a function of the center frequency used for the experiment, again increasing substantially as the center frequency increases.

The two experiments which were reported here provide an interesting case in point. The values of \( z_0 \) for both blood and tissue mimicking material turn out to be almost identical. It is only the value of \( n \) which differs, and even there the difference doesn't appear large. The errors which are encountered in using the linear assumption for those cases, however, differ markedly. At a center frequency of 4.8 MHz, the use of linear analysis for blood results in an \( \epsilon \) value of 37\%. When \( n \) increases to that of tissue mimicking material, and all other parameters remain essentially the same, the corresponding \( \epsilon \) rises to 130\%. Thus, in order to be able to compare results from different laboratories, which have been obtained under different experimental conditions such as center frequency, it becomes imperative that nonlinear analysis be used, unless the special case of linearity has been proved a priori.

The two parameter description of the material attenuation obtainable through nonlinear analysis is achieved at the expense of increased experimental and computational complexity. In the linear case, one measurement of \( \Delta f \) is sufficient to obtain \( z_0 \), which is the sole parameter needed to describe the attenuation of the material. In the nonlinear case described by eqns (4) and (5), \( z_0 \) and \( n \) are the two unknowns. For this reason at least two independent measurements must be made at two different bandwidths or different center frequencies. Due to the nonlinearity of the equation, however, multiple measurements have to be made in order to solve eqn (5) for \( n \) and \( z_0 \) (Narayana and Ophir, 1983).

REFERENCES