A Narrowband Pulse-Echo Technique for *In Vivo* Ultrasonic Attenuation Estimation

J. OPHIR, R. E. McWHIRT, N. F. MAKLAD, AND P. M. JAEGGER

*Abstract*—The principles of a narrowband technique for pulse echo attenuation measurements are presented. Experiments are described which compare the attenuation in tissue-mimicking phantoms, measured by known methods, to the attenuation measured by the present technique. The effects of various experimental parameters on the results are investigated. The attenuation in spleens of canine models and in livers of human volunteers is presented.

I. INTRODUCTION

Of the many methods available for the measurement of ultrasonic attenuation [1], only a few are suitable for *in vivo* measurements. The standard pulse echo methods, such as substitution methods that involve the use of standard reflectors, are not suitable for such applications. Transmission methods [2]–[4] are similarly not suitable for attenuation measurements for organs other than the breast.

A number of methods to estimate attenuation from backscattered echoes has been developed in the past several years [5]–[7], [9]. These can broadly be divided into time domain and frequency domain methods, which are related by the Fourier transformation. In these two domains, both amplitude changes and center frequency shifts experienced by the incident pulse contain information about tissue attenuation. By extracting the attenuation information from amplitude estimates, both wideband and narrowband approaches can be utilized. The frequency domain wideband approach, known as the spectral difference method [23], involves estimating the mean log spectra of backscatter echoes at two depths in the tissue, and then subtracting them. This difference versus frequency is an estimate of the frequency dependence of attenuation over the bandwidth used. Note that since no assumption is made about the magnitude of the attenuation at zero frequency, the estimate is only a *relative* attenuation slope estimate. Moreover, a bandwidth compromise must be made: if the bandwidth of the measurement is small, large errors in the slope of the attenuation versus frequency are possible; if the bandwidth is large, spectra with higher signal-to-noise ratios are encountered on either side of the center frequency, again resulting in large errors. Thus, this technique is prone to large bias errors [24]. Moreover, it is computationally slow.

A real-time wideband amplitude technique, described by Mountford and Wells [15], estimates the amplitude of backscatter echos as a function of depth. Because of the wideband nature of the pulse used, its center frequency does not stay constant, but rather experiences a frequency downshift which is dependent on the square of the bandwidth [5], [2]. It can be shown [2] that this effect results in the decay of energy with depth which is exponential, but not as a linear function of the attenuation and depth, but rather with an additional quadratic component proportional to the square of the product of the attenuation coefficient, depth, and bandwidth. In addition to these problems, both of the above techniques are highly sensitive to the transducer axial beam pattern, which, in turn, is a function of the (unknown) tissue attenuation and scattering properties.

A narrowband, pulse echo attenuation estimation technique was described by Ophir *et al.* [8], [14], [20]. This technique overcomes the bias errors inherent in the spectral difference methods by estimating the *absolute* attenuation coefficient; the quadratic term inherent in the wideband amplitude technique is essentially eliminated by the use of narrowband acoustic emission; finally, the beam profile artifacts are greatly reduced by using a differential, constant depth C-scan method, which has certain similarity to an earlier transmission technique reported by Carstensen [25].

This paper discusses the basic principles of operation of the narrowband technique, along with its hardware implementation. A number of experiments is described which were carried out in order to investigate the accuracy of the attenuation estimates. The effects of the bandwidth of the transmitted pulse, the width of the transducer beam, the separation between samples vis à vis the transducer beamwidth, and the effect of overlying layers were investigated. Finally, the attenuation in *in vivo* canine models and human livers is presented.

II. THEORY

A technique for the quantitative estimation of the bulk ultrasonic attenuation of the homogenous material which backscatters ultrasound was introduced by Ophir and Maklad [8]. The method is essentially a differential, narrowband C-scan technique. Subsequent work has extended this approach to the B plane and to multiple frequencies [14]. A focused transducer is excited with narrowband pulses. The transducer is immersed in a standoff water bag and is laterally translated while demodulated echo amplitude data are recorded from two different ranges along the ultrasonic beam axis, thus

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defining two constant depth planes in the target material. From the echo data, the first and second moments of the backscattered log amplitude distribution are computed for each plane. The differences in the two means divided by twice the plane separation is an estimate of the attenuation of the material at the center frequency of the ultrasonic pulse. The error in the estimate is a function of sample variances, the number of samples taken from each plane, the separation between planes and the difference in the transducer’s beam sensitivity profile between the two ranges.

From Reid [10], the ratio between the received and transmitted electrical power for backscatter from a scatterer in the far field of and ultrasound transducer can be expressed as

\[
P_r(R) = \frac{T^2\alpha_b A^2 \exp(-4\alpha R)}{R^4\lambda^2}
\]

where \( R \) is the distance from the transducer to the scatterer, \( \lambda \) is the wavelength assuming monochromatic illumination, \( T \) is the efficiency of the transducer, \( A \) is the effective aperture of the transducer, \( \alpha \) is the attenuation coefficient of the target at frequency \( f_o = c/\lambda \) (where \( c \) is the speed of sound propagation in the target), and \( \alpha_b \) is the backscatter cross section. If the medium between the transducer and the scatterer is composed of attenuating target and nonattenuating (water stand-off) parts, then (1) can be rewritten as

\[
P_r(R) = \frac{T^2\alpha_b A^2 \exp(-4\alpha d)}{R^4\lambda^2}
\]

where \( d = R - W \) is the portion of \( R \) for which \( \alpha \) is nonzero, and \( W \) is the portion for which \( \alpha \) is zero. If we assume that there are two scattering ensembles at ranges \( R_1 \) and \( R_2 \), respectively, having the same value for \( \alpha_b \) and both within the attenuating target, then the ratio of the received electrical powers from both ranges is

\[
\frac{P_r(R_1)}{P_r(R_2)} = \exp\left[\frac{4\alpha(d_2 - d_1)}{(R_1/R_2)^4}\right]
\]

where \( d_1 = R_1 - W \) and \( d_2 = R_2 - W \). The constant speed of sound in the water is assumed to be the same as that of the tissue. The wavelength is also assumed constant [see discussion following (10)]. The numerator of (3) contains the desired attenuation information, whereas the denominator is a beam-spreading loss which needs to be experimentally eliminated or corrected for.

Elimination of this term altogether can be achieved by performing the measurement such that the acquisition of the data is done in two steps: 1) data are acquired from a given range \( R = R_1 = R_2 \) first, and then 2) the transducer is moved axially in the water by a known amount \( |d_2 - d_1| \) and data are again taken at the same range \( R \) from the transducer, but which is now at a new depth in the target. The preservation of the range eliminates the spreading loss term. Thus, if \( R_1 \) and \( R_2 \) are equal, but \( d_1 \) and \( d_2 \) are not, then (3) becomes

\[
\frac{P_r(d_1)}{P_r(d_2)} = \exp\left[4\alpha(d_2 - d_1)\right].
\]

Noting that \( P_r \) is proportional to the square of the received voltage \( V_r \), and solving for \( \alpha \) in dB/cm, yields

\[
\alpha = 1.16 \left[ \frac{\log_{10} \frac{V_r(d_1)}{V_r(d_2)}}{d_2 - d_1} \right]
\]

Generalizing (5) by assuming that measurements are made on many independent scatterers, the estimated value for the attenuation of the medium becomes [11]

\[
\bar{\alpha}(\text{dB/cm}) = 1.16 \left[ \frac{\log_{10} \frac{V_r(d_1)}{V_r(d_2)}}{d_2 - d_1} \pm t_{0.025} \frac{s^2(d_1) + s^2(d_2)}{n(d_1) + n(d_2)}^{1/2} \right]
\]

where the bar denotes average value, and

\[
s^2(d_1) = \text{sample variance at depth } d_1,
\]

\[
s^2(d_2) = \text{sample variance at depth } d_2,
\]

\[
n(d_1) = \text{number of independent samples at depth } d_1,
\]

\[
n(d_2) = \text{number of independent samples at depth } d_2,
\]

\[
t_{0.025} = \text{student’s } t \text{ critical point for the 95 percent confidence interval, with } 2n - 2 \text{ degrees of freedom.}
\]

Typically, the number of degrees of freedom is greater than 250, and thus, \( t_{0.025} \approx 1.96 \) [11].

Note that all statistics are performed on the logarithm of the received voltages. Assuming that the backscatter process is of equal variance at both ranges, or \( s^2(d_1) \approx s^2(d_2) = s^2 \), and that \( n(d_1) = n(d_2) = n \), we write

\[
\alpha = 1.16 \left[ \frac{\log_{10} \frac{V_r(d_1)}{V_r(d_2)}}{d_2 - d_1} \right] + t_{0.025} \frac{s\sqrt{2/n}}{d_2 - d_1}
\]

The first term in this expression is the estimate of the difference in the mean log signal amplitudes per unit length, which is essentially a constant for any particular target material and frequency. The second term is the error in the estimate. Since the total scanned volume between the planes is proportional to \( n(d_2 - d_1) \) (assuming an equal and constant separation between adjacent samples in the plane), while the error decreases as \( n^{1/2}(d_2 - d_1) \), it is concluded that for a given scanned volume, the best error performance will be achieved by maximizing the separation between the planes and minimizing \( n \). This is strictly true only for large values of \( n \). There is a case, however, where the quantity \( (d_2 - d_1) \) is increased and \( n \) is proportionally decreased (and thus maintaining a constant scanned volume) to a point where \( t_{0.025} \) itself starts increasing rapidly [11], thus actually increasing the error magnitude. This becomes important (>10 percent increase from the \( n = \infty \) case) only at the point where the number of degrees of freedom \( 2n - 2 < 14 \), or \( n < 8 \).

The technique assumes that the excitation is essentially monochromatic, implying zero bandwidth. This, however, is only approximated by the finite duration pulse. It is known
that a Gaussian pulse which propagates in the lossy medium
with a linear frequency dependence of attenuation suffers a
center frequency downshift

\[ \Delta f = 4 \alpha_0 \sigma^2 |d_2 - d_1| \]  \hspace{1cm} (8)

where \( \alpha_0 = \bar{a}/f_o \) is the amplitude attenuation frequency
dependence coefficient, \((d_2 - d_1)\) is the distance between the
planes, \( f_o \) is the center frequency of the transducer, and \( \sigma^2 \) is
the variance of the spectrum where \( \sigma^2 = 0.18B^2 \) and \( B \) is the
half amplitude bandwidth \([21]\). Since the attenuation itself is
frequency dependent, a downshifted center frequency may result
in an artificially low estimate of the attenuation.

In order to investigate this effect, we compute the expected
downshift in frequency. It has been shown by Narayana and
Ophir \([17]\) that for a rectangular burst excitation of duration
\( T_o \) with a sinc (-) spectrum, under the condition \( \pi T_o \Delta f \ll 1 \),
we can identify the variance of an equivalent Gaussian spec-
trum as

\[ \sigma^2 = 2/\pi^2 T_o^2 . \]

Thus, we can write (8) as

\[ \Delta f = 8 \alpha_0 |d_2 - d_1|/\pi^2 T_o^2 . \]  \hspace{1cm} (10)

Typical values are

\[ \alpha_0 = 0.06 \text{ cm}^{-1} \cdot \text{MHz}^{-1} \text{ for soft tissue} [5] \]
\[ d_2 - d_1 = 2 \text{ cm} \]
\[ T_o = 2 \mu s, \text{ corresponding to 6-7 sine wave cycles at } f_o \]
\[ f_o = 3 \text{ MHz} . \]

Substituting in (10) we get \( \Delta f = 24 \text{ kHz} \) (and thus the condition
\( \pi T_o \Delta f < 1 \) holds true), or \( \Delta f/f_o = 0.8 \text{ percent} \). Clearly,
the spectrum of 2 \mu s burst suffers a very small frequency downshift.
Even a thick slab of tissue (\( \geq 10 \text{ cm} \)) causes a frequency
downshift on the order of only 4 percent of the center fre-
cquency of the transducer. Thus, the changes in the value of
estimated attenuation itself \( \bar{a}(f_o) \) are expected to be minimal
due to the small downshift in frequency, i.e., \( \alpha_0 f_o \approx \alpha_0 (f_o - \Delta f) \) for \( \Delta f < < f_o \).

Complete decorrelation of samples along the axis of the
transducer is assured by using plane separations on the order of
1-2 cm, while using pulse durations which are on the order of
2 \mu s or less which correspond to about 3 mm tissue thick-
ness. In the lateral dimension, however, care must be taken
to provide enough spacing between adjacent sample points.
This can be done by ensuring that the sample spacing is at
least as large as half the width of the \( J_1(x)/x \) directivity func-
tion of the transducer beam at the appropriate range in the
far field of the transducer \([16]\). This, of course, places a
lower limit on the spacing between samples for a given trans-
ducer, and ultimately on the minimum scanned volume.

III. MATERIALS, METHODS, AND RESULTS

To determine the utility of the technique, an apparatus was
developed and a series of experiments was performed.

A. Apparatus

Fig. 1 shows a block diagram of the apparatus. The tone
burst generated by the function generator is amplified by a
linear, wideband, power amplifier and is used to pulse the
transducer while the computer-controlled motorized scanner
performs a rectilinear scan. The transducer is immersed in a
water bag mechanism to provide acoustical coupling during
the scans and to allow for the axial translation of the trans-
ducer between C-scan planes if the beam profile is not corrected
for. The received backscatter signals from the target are
logarithmically amplified and demodulated. A computer-
controlled variable digital delay circuit triggers a fast sample
and hold device, which acquires amplitude samples from a
desired depth in the target. These samples are then digitized
by an 8-bit A/D converter, and are stored in a buffer memory.
After the completion of the first C scan, the computer triggers
a mechanism to rapidly translate the transducer along the
beam axis into position for a second C scan. At the end of the
scans, the computer is used to perform the calculation of the
attenuation coefficient using (6). Typical amplitude histo-
grams are shown in Fig. 2.

B. Effect of Center Frequency

Pulse echo substitution measurements \([1]\) were first made
on various kinds of tissue-mimicking materials (TMM) \([12]\)
at various frequencies, at 22°C, to independently determine
the attenuation properties of the material. A 3.5 MHz wide-
band transducer was operated in the pulse echo mode. The
transducer was excited by a 4-\mu s-long tone burst. The pulse
propagated in distilled water and the echo from a thick stain-
less steel reflector at normal incidence was recorded. A 2-cm-
 thick sample of TMM was interpolated between the transducer
and the reflector, and the new echo compared to the recorded
echo. The amplitude difference between the echoes was con-
sidered to be due to the bulk attenuation of the TMM at the
center frequency of the pulse. Reflections at the TMM-water
interfaces were negligible. Measurements were taken at center
frequencies of 2.25, 3, 3.5, and 4 MHz, and a least-squares
power law fit to the data was carried out. The best fit was
determined to be

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frequencies of 2.25, 3, 3.5, and 4 MHz, and a least-squares
power law fit to the data was carried out. The best fit was
determined to be
\[ \alpha(f) = 0.30 f^{1.38} \text{ dB/cm}^{-1} \cdot \text{MHz}^{-1.38} \]

with a coefficient of determination \( r^2 = 0.99 \) (\( r^2 = 1 \) indicated a perfect fit).

Once the attenuation of the TMM was determined, a series of C-scan measurements was taken from the TMM in distilled water at the focus of the transducer. A 3.5 MHz, 19 mm aperture focused transducer was used to measure the attenuation at 3, 3.5, and 4 MHz, using 2 cm plane separation and target range of 9 cm; and a 5-MHz 13-mm focused transducer was used to measure the attenuation at 4.5, 5, and 5.5 MHz at a range of 5.75 cm using 1 cm plane spacing. The number of samples was 500 samples per plane and the separation between samples was 2.1 mm.

Results are shown in Fig. 3. Note that the power law fit to the results obtained by substitution is in good agreement with the results obtained by the use of the narrowband pulse echo method over a finite range of frequencies.

C. Effects of Overlying Layers

The differential nature of the technique suggests that the measurement of attenuation should be independent of the effects of overlying layers. This, of course, is an important consideration when in vivo measurements are undertaken, where the region which is to be measured resides deep within the body. Two experiments were undertaken in order to test the effects of overlying layers.

1) Experiment I: Fig. 4 illustrates the experimental arrangement used to simulate a large discontinuity in the thickness, and thus attenuation, and speed of sound of the overlying material. A tissue-equivalent phantom was used [13]. The phantom consisted of TMM covered in a Plexiglas\textsuperscript{®} box. A \( \frac{1}{16} \) in Plexiglas\textsuperscript{®} window formed the top of the box. The attenuation and speed of sound in the TMM was approximately...
that of normal human liver in vivo. The phantom was completely submerged in distilled water and then scanned so that the axis of the transducer beam remained normal to the Plexiglas® window of the phantom during the C scans. The focal point of the beam during C scan was 3 cm below the window. The focal length of the 5.08 cm diam transducer was 25 cm and the 6 dB beamwidth at the focus was 2.1 mm. The center frequency of the 3 µs pulse burst was 3.5 MHz. Sample spacing was 2.1 mm, and C-scan plane separation was 2 cm. The number of samples per plane was 1000. The approximate dimensions of the C-scan planes was 10.5 by 4.2 cm. In case I of Fig. 4, the phantom was scanned through the standard 0.16 cm window. In case II of Fig. 4, one half of the scan passed through the standard window, and the other half through a 0.79-cm-thick Plexiglas® window. The ratio of the speed of sound in Plexiglas® to that in the phantom material was approximately 1.7. Results are given in Table I. It can be observed that the introduction of the discontinuous window has no appreciable effect on the results.

2) Experiment II: This experiment is illustrated in Fig. 5. The arrangement was used to simulate the continuous change in the thickness of the overlying material during the scan and the effects of nonnormal specular reflections from the window of the phantom. Case I conditions for Fig. 5 were identical to those of case I of Fig. 4. In case II of Fig. 5, the phantom was angled 4.5° to the transducer beam and the measurement repeated. This represented a 0.8 cm change in the thickness of the overlying material within the scan. In case III of Fig. 5, the measurement was repeated with the phantom angled at 11°, representing a 2 cm change in thickness. Results were given in Table II.

D. Effects of Finite Bandwidth

Attenuation measurements were performed on TMM at 3.5 MHz, using 1.5 mm sample spacings, 2 cm plane spacings, and 500 samples/planer The number of sine wave cycles in the 3.5 MHz tone burst was sequentially changed from 1 to 8. Since the “effective” half amplitude bandwidth \( B = 1.06/T_0 \) [17], this range corresponds to excitation bandwidths ranging from 3.7 to 0.46 MHz, respectively. The transducer, however, had a fractional bandwidth of 25 percent, or 0.88 MHz, which was the limiting factor in this experiment. The total combined bandwidth of the excitations and the transducer was computed in Appendix I and is shown in Fig. 6. It is observed from the figure that the effect changing the bandwidth from 12 percent to 25 percent has a small effect on the measurement. Larger bandwidths, however, will be expected to result in larger errors in the measurement [9].

E. Effect of Sample Spacing

An experiment was performed in order to investigate the effect of reducing the spacing between adjacent samples in the plane in relation to the beamwidth. The pulse echo beamwidth of the focused 50 mm aperture, 3.5 MHz circular transducer at the focus (25 cm), was measured in the water tank.
using a 0.16 mm diam steel ball target. The beamwidth corresponding to the level which was 3 dB below the maximum sensitivity was 1.5 mm, while the 6 dB beamwidth was 2.1 mm. Measurements of attenuation were then undertaken in TMM, whereby the spacing between samples was altered, but the total number of samples remained constant. The results are shown in Fig. 7. Note that the mean attenuation remained essentially constant up to sample spacing which were separated by one half the 3 dB beamwidth.

F. In Vivo Experiments

Preliminary in vivo experiments were performed on canines and human volunteers. Typical scanning parameters were 0.9-1.5 mm sample spacing in the C-scan plane, 1 or 2 cm plane separation, 500-1000 samples per plane, ultrasonic pulse duration of 2 μs with a pulse center frequency of 3.5 MHz, and a focal length of 9 or 25 cm. Water bag temperature was 37°C.

In the canine experiments, the animals were anesthetized and the spleen was surgically exposed and immobilized. The water bag was placed in contact with the spleen, C-scan was performed at a plane separation of 1 cm, and the attenuation determined. The attenuation was independently verified in one case using a through-transmission substitution method in situ. This was done by coaxially mounting a short focus 19 mm circular transducer and a wideband 1 mm² aperture hydrophone on the opposing ends of a special C-clamp arrangement. The transducer was used on a transmitter and the hydrophone as a receiver. The C-clamp was positioned over the spleen until good acoustic contact with both sides was obtained. The amplitude of the narrowband signal from the hydrophone was recorded. The clamp was then removed and immersed in saline, and another amplitude measurement was made. The attenuation coefficient was estimated from (5). These measurements were repeated 10 times. Table III summarizes three canine spleen experiments. In spleen (C) the values were compared to those determined using the transmission technique, and good agreement was found.

In the human experiments, the water bag was placed in contact with the abdomen, and the liver was scanned while the subject held his breath. Typical scanning time was 20 s. The values obtained from normal livers was consistent with literature values. Numerous measurements of the attenuation in in vivo quadriceps femoris muscles were also made and are reported elsewhere [20].

IV. SUMMARY AND DISCUSSION

In this paper, we have investigated the conditions under which accurate estimates of the attenuation in vivo and in vitro can be made using a narrowband pulse echo method. The assumptions underlying this technique have been investigated theoretically and/or experimentally as follows.

1) In homogeneous materials such as TMM's, liver, and spleen, the use of (7) is justified; i.e., the difference in the log of the mean amplitudes from two planes divided by the plane separation is a good estimate of the attenuation coefficient over some band of frequencies. It is not, however, expected that this equation will hold in inhomogeneous materials, where the average scattering cross section and/or the scattering law change from one location to another in the target materials. Similar restrictions apply to other pulse echo attenuation measurement techniques. An example is given by Flax et al. [19].

2) The effect of overlying material has been experimentally shown to be negligible. We have obviously not simulated all possible configurations of such materials, but it appears that at least in some extreme cases the measurement is independent of the intervening layers. In principle, this is expected to the first approximation since differential measurements are usually relatively free from common mode effects, i.e., the overlying layers have a similar effect on the statistics of the echoes from both planes.

3) Effects due to beam profile variations can be essentially eliminated by axial translation of the transducer such that the plane of interest remains at a constant range. This, of course, is only possible if a water bag mechanism is available.

4) This technique can be extended to the B plane (the plane which contains the transducer beam axis) by taking more differential samples from points which are close together. This has been done in [14] and was used clinically in diffuse liver disease [22]. However, methods for correcting for beam profile effects must then be incorporated.

5) The effects of finite bandwidth and sample spacing has been shown to be within the theoretical limits. They do, however, ultimately affect the minimum size of the target region which can be measured; highly focused beams and short pulses allow smaller regions to be measured. In the limit, however, the CW assumption will break down and depth of field limitations will start to play a more important limiting role.

6) We have shown that the technique can be extended to multiple measurements over a band of frequencies, although the time and computational advantage is lost if this is done sequentially. When done in parallel, however, this advantage is maintained [14].

7) Fig. 3 demonstrates that bias errors are essentially nonexistent over a wide band of frequencies, and that the absolute value of the attenuation coefficient can, in fact, be estimated. This is in contrast to other estimation techniques where only the slope of the attenuation coefficient versus frequency can be estimated [5], [7], [24].
TABLE III

<table>
<thead>
<tr>
<th>Organ</th>
<th>C Scans</th>
<th>95 Percent Confidence Interval</th>
<th>Number of Experiments</th>
<th>Independent Value and Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dog Spleen (A)(^a)</td>
<td>1.52 ± 0.33</td>
<td>7</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>Dog Spleen (B)(^\ast)</td>
<td>1.27 ± 0.09</td>
<td>19</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>Dog Spleen (C)(^a)</td>
<td>1.56 ± 0.13</td>
<td>21</td>
<td>1.41 ± 0.20 (from 10 through-transmission measurements on same organ)</td>
<td></td>
</tr>
<tr>
<td>Human Liver (1)(^c)</td>
<td>1.67 ± 0.31</td>
<td>10</td>
<td>1.58-1.93 [5]</td>
<td></td>
</tr>
<tr>
<td>Human Liver (2)(^c)</td>
<td>1.77 ± 0.24</td>
<td>10</td>
<td>1.58-1.93 [5]</td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\)Surgically exposed.
\(^{\ast}\)Placed on platform, intact.
\(^{c}\)Normal liver.

TABLE IV

<table>
<thead>
<tr>
<th>Number of Cycles at 3.5 MHz</th>
<th>Effective Excitation Bandwidth (B_E) MHz</th>
<th>Transducer Bandwidth (B_T) MHz</th>
<th>Total Combined Bandwidth (B) MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.66</td>
<td>0.88</td>
<td>0.86</td>
</tr>
<tr>
<td>2</td>
<td>1.83</td>
<td>0.88</td>
<td>0.79</td>
</tr>
<tr>
<td>3</td>
<td>1.22</td>
<td>0.88</td>
<td>0.71</td>
</tr>
<tr>
<td>4</td>
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<td>0.88</td>
<td>0.63</td>
</tr>
<tr>
<td>5</td>
<td>0.73</td>
<td>0.88</td>
<td>0.56</td>
</tr>
<tr>
<td>6</td>
<td>0.61</td>
<td>0.88</td>
<td>0.50</td>
</tr>
<tr>
<td>7</td>
<td>0.52</td>
<td>0.88</td>
<td>0.45</td>
</tr>
<tr>
<td>8</td>
<td>0.46</td>
<td>0.88</td>
<td>0.41</td>
</tr>
</tbody>
</table>

APPENDIX

The product of two Gaussian spectra of identical center frequency and variances \(\sigma_1^2\) and \(\sigma_2^2\) is another Gaussian whose center frequency is unchanged and its variance is given by [18]

\[
\Sigma^2 = \sigma_1^2 \sigma_2^2 / (\sigma_1^2 + \sigma_2^2). \tag{A1}
\]

In this case, the bandwidth of the transducer's transfer function is assumed to be a Gaussian centered at 3.5 MHz with a 6 dB bandwidth \(B_T = 0.88\) MHz. The effective bandwidths of the excitation pulses \(B_E\) (1–8 cycles) are given in Table IV. The total combined 6 dB bandwidth of the excitation and the transducer can thus be written as

\[
B = B_T \frac{B_E}{(B_T^2 + B_E^2)^{1/2}}. \tag{A2}
\]

In the limit when \(B_E \gg B_T\), \(B \approx B_T\), and when \(B_E \ll B_T\), then \(B \approx B_E\). Table IV shows the effect of the transducer bandwidth on the total combined bandwidth. The values of \(B\) are also shown in Fig. 6.

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