ELIMINATION OF DIFFRACTION ERROR IN ACOUSTIC ATTENUATION
ESTIMATION VIA AXIAL BEAM TRANSLATION

J. Ophir and D. Mehta

Ultrasound Laboratory
Department of Radiology
University of Texas Medical School
Houston, TX 77030

Optimized wideband attenuation estimations were performed on a tissue mimicking phantom in a water tank with and without axial beam translation (ABT), and the results were compared to those from standard substitution measurements. A -17 percent discrepancy between the results of the substitution method and the optimized estimation without ABT was noted in the far field. This discrepancy was eliminated when ABT was utilized. © 1988 Academic Press, Inc.

Key words: Attenuation; axial beam translation; beam correction; diffraction; estimation; spectrum; tissue characterization; ultrasound.

I. INTRODUCTION

In recent years a number of techniques have been developed for estimating the attenuation of soft tissues from reflected ultrasound, which have been reviewed by Ophir et al [1]. In particular, a "spectral difference" method was proposed [2,3], whereby spectra from varying depths in tissue are acquired. The log-spectral differences are then computed and plotted against frequency. A linear or power-law function is fit to the data and the behavior of attenuation vs. frequency is derived. This technique allows the use of rather arbitrary ultrasonic excitation spectra, and would in principle work in tissues where the frequency dependence of attenuation is linear or nonlinear. Thus, this technique is a practical one, and broadband [4,5] and narrowband [6] implementations have been used in vivo with some success. Attenuation estimations using this (and other) techniques tend to suffer from bias errors. These errors are attributable to two principal mechanisms:

(1) The details of the signal processing algorithms used in attenuation estimations can bias the estimate [7]. The ultrasonic echo sequence is normally broken up into temporal windows for which spectra are computed. Bias errors in the estimate of the attenuation coefficient are encountered when the window length is suboptimal. The degree of window overlap may also contribute to these errors. The bias in the estimate as a function of bandwidth is another parameter that is important. We have found that the bias errors could be substantially reduced by an optimal combination of these parameters.

(2) An underlying assumption in the use of all spectral estimation techniques is that the impulse response of the transducer is range independent. This clearly is a simplistic view, since significant range dependence of the impulse response is known to exist [8]. Early workers in this area have ignored the effects due to variations of the impulse response with depth [9, 10]. Ophir et al [1,11,12] have used a system whereby transducer axial beam translation (ABT) was used in conjunction with a narrowband spectral difference method in an effort to reduce or eliminate
the effects due to beam diffraction, but such improvements have not been quantitated. Empirical corrections using plane reflectors and tissue mimicking phantoms were also reported [13,14,15,16,17]. However, Robinson et al [18] have demonstrated that the apparent pulse-echo beam response of the transducer as a function of range varies significantly for different reflectors, and thus the correction to be applied to the data varies according to the type of (generally unknown) tissue being examined. They further demonstrated that using tissue mimicking phantom material to establish "beam correction" in their system gave rise to variations of up to 0.25 dB cm\(^{-1}\) MHz\(^{-1}\) (about 50% of the value of attenuation of normal liver! [19]) in different parts of the beam. The fact that pulse-echo beam characteristics are clearly different in different tissues suggests that it may not be possible to design a universal beam correction function which will give satisfactory results in unknown tissues.

In this paper, we investigate the applicability of the axial beam translation method (ABT) to the more generalized wideband spectral difference technique. The method relies on collecting ultrasonic echo data emanating from successively deeper regions in the target, but which are at a constant range from the transducer aperture. This method requires scanning with a water offset mechanism, which may be incompatible with standard clinical contact scanning. However, it does not require calibration, and renders unbiased estimations of attenuation in unknown tissues. We have conducted a series of attenuation estimation experiments in well characterized tissue mimicking material in a water tank. The slope of attenuation vs. frequency (\(\gamma\)) was estimated in the near field and the far field of an unfocussed transducer with and without ABT, and was compared with corresponding values obtained using a classical substitution method under the same conditions. A substantial reduction in diffraction bias errors due to the use of ABT was found.

II. MATERIALS AND METHODS

Two sets of experiments were conducted on a slab of reticulated polyester foam target in water [6]. The first set of experiments was conducted on the target in order to establish its attenuation coefficient slope, using a classical substitution method. The second set of experiments involved using the spectral difference technique, with and without axial beam translation under various conditions, to estimate the attenuation coefficient slope in the same foam target. The results of the two techniques were then compared. All signal processing operations were performed in software, using a 512 point Fast Fourier Transform of echo segments windowed by a 10% cosine apodization function.

A. Substitution Method

The slope of attenuation vs. frequency of a 20 pore-per-inch, 5.0 cm thick 12 cm diameter circular foam pad was measured after proper preparation [6], using a wideband substitution technique. The transducer (Echo Labs, Model E87x481) had a center frequency of 3.5 MHz, and an aperture of 19 mm (unfocussed). The measurement bandwidth was 2.50-4.75 MHz. Near (7 cm) and far (23 cm) fields were examined.

The experimental setup is shown in figure 1. The transducer was placed near the top of a 60 gallon water tank maintained at 37.0 \(\pm\) 0.5 \(\degree\)C. A 1.27 cm thick plexiglas plate was placed under the foam target and was used as a reference planar reflector. The reflector was aligned for normal wave incidence.
The electronic apparatus is controlled by a Compaq-286 personal computer via an IEEE-488 bus system. The stepper motor controller (Superior Electric Co.) enables the movement of the transducer in multiples of 2.5\,\mu m increments. The transducer is shock excited by the transmitter (Metrotek Corp.) at a rate of 1-2 kHz. The received signal is amplified by an input protected custom preamplifier and fed into a computer controlled attenuator and rf amplifier. The attenuator works in an automated interactive software driven mode, so that the peak amplitude of the signal in a given time window of interest always occupies the input range required by the LeCroy digitizer in order to utilize its full 8 bit resolution capabilities. The appropriate attenuator settings are later used to compute the absolute magnitude of the signal. In the present experiments, the digitization rate was set at 25 MHz.

A total of 16 echoes from the plexiglas plate were obtained at transducer positions separated by 4 mm in the absence of the foam. These 16 positions were adequate, since the echo from the plate barely changed when the transducer was moved. This is demonstrated by the smooth spectra of figure 2. The procedure was repeated with a piece of foam interposed between the transducer and the reflector. The spectra of the 16 echoes were averaged in both cases to give two average spectra. The log spectral difference of the two spectra was computed, and the frequency dependence of attenuation was derived from the slope of a linear least squares fit of the log spectral difference versus frequency. Pairs of averaged spectra from the near field and far field of the transducer are shown in figure 2a, b.

B. Estimation Methods

Estimation of \( \beta \) was performed using ultrasonic signals acquired in two modes using the apparatus of figure 1:

Mode 1 - This is the standard data acquisition mode, whereby the distance between the transducer aperture and the target remains constant. In our
Fig. 2 Spectra from planar reflector obtained in the substitution experiments. (a) near field. (b) far field. Curves A are spectra without foam. Curves B are spectra with foam. Magnitudes less than -40 dB have been eliminated.

experiment, the distance between the aperture and the distal edge of the target was either 7 cm or 23 cm. A software range gate was used to window the desired signal segments. This mode is shown in figure 3 a,b.

Mode 2 - This is the ABT mode, which involves translation of the transducer along its radiation axis, while the distance between the transducer aperture and the range gate remains constant. In our experiments, this distance was either 7 cm or 23 cm. Transducer translation was done in 8 mm steps. This mode is shown in figure 3 c,d.

Fig. 3 Standard estimation experiments. (a) near field without ABT. (b) far field without ABT. (c) near field with ABT. (d) far field with ABT.

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Fig. 4 Lateral correlation function of whole demodulated echo sequences in foam. Sequences were acquired at a lateral distance of 1 mm from each other. It can be seen that the correlation length is $\sim 4$ mm.

A total of 240 A-lines were acquired in each mode. This number of A-lines represents the approximate number of uncorrelated A-lines in the scanned area. The scanned area in the foam was 8x8 cm. Since the lateral correlation length of the echo sequences was determined to be approximately 4 mm (Fig. 4), 5 mm spacings between adjacent echo sequences were used to assure the absence of correlation. All data from the 320 cm$^3$ volume was used in the estimation.

The spectral difference technique was implemented in two steps:

1. For each discrete frequency within the desired bandwidth, a linear fit was obtained for the magnitude of the average spectra (dB) versus depth (cm).

2. A second linear fit was obtained for the various slopes of the lines from part (1) (dB/cm) versus frequency (MHz) to determine the attenuation coefficient slope (dB/cm/MHz). The method used to compute the standard deviations is shown in Appendix A. Bias errors due to the signal processing parameters listed below were minimized before performing the linear regression. The signal processing parameters that were found to influence the bias in the estimate are window length, degree of adjacent window overlap, and the bandwidth of the measurement.

The separate dependences of the attenuation coefficient estimate on these parameters were studied in the following manner:

1. Window length
   Keeping the bandwidth and the degree of window overlap constant, the window length was changed from 0.4 to 2 cm in 0.2 cm increments and the concomitant variation in the value of the estimated attenuation coefficient was observed. The optimal window length which minimizes the bias error was determined.

2. Degree of window overlap
   For the optimal window length, the degree of window overlap was varied from 0 to 50 percent in increments of 1 percent, and from 50 to 90 percent in increments of 10 percent. The optimal percent overlap where the bias error was the minimum was determined.

3. Bandwidth
   Having found the optimal window length and percent window overlap, the dependence of the bias error on the bandwidth was studied in two steps:
(a) For a given starting frequency, e.g., 2.5 MHz, the ending frequency was varied between 4.45 to 4.8 MHz in 0.05 MHz increments and the parameter $\beta$ determined for each bandwidth.

(b) The starting frequency was varied from 2.5 to 3.2 MHz in 0.1 MHz increments and step (a) was repeated for all these frequencies.

From (a) and (b), the optimal starting and ending frequencies were determined such that the bias error was minimized. For steps (1) through (3) above, the bias error in percent was determined as

$$\epsilon_{\text{bias}} = \frac{\beta_{\text{sub}} - \beta_{\text{est}}}{\beta_{\text{sub}}} \times 100\%$$  \hspace{1cm} (1)

where $\beta_{\text{sub}}$ = the attenuation frequency slope determined from the substitution measurements

$\beta_{\text{est}}$ = the attenuation frequency slope determined from the respective estimation procedure.

III. RESULTS

A. Substitution Method

The results of the measurements of the frequency slope of attenuation using the substitution method in the near field and in the far field of the transducer are shown in Table 1. The averaged spectra from the plexiglas reflector in the presence and absence of the foam are shown in figure 2a (near field) and figure 2b (far field).

B. Estimation Method

Averaged spectra (over 240 echo sequences) from 6 progressively farther windows in the far field with and without ABT are shown in figure 5a,b. The averaged spectra from 6 progressively farther windows in the near field with and without ABT are shown in figure 6a,b.

![Fig. 5](image)

Spectra from estimation experiments in 6 regions separated by 8 mm depth increments in the far field. (a) without ABT. (b) with ABT. Magnitudes less than -40 dB have been eliminated.
Fig. 6 Spectra from estimation experiments in 6 regions separated by 8mm depth increments in the near field. (a) without ABT. (b) with ABT. Magnitudes less than -40 dB have been eliminated.

The optimization of the signal processing is shown in figures 7 through 9. The effect of window length on the estimate in the near field and the far field is shown in figures 7a,b. It can be seen that the estimate is indeed dependent on window length, reaching a minimum bias error when the window length equals 1.2 cm. Also, it can be concluded that the near field is plagued by larger bias errors than the far field. For this optimal window length, the effect of window overlap is shown in figure 8a,b. It can be observed that an overlap of approximately 46% results in a minimum bias error. The effect of bandwidth on the bias error was determined using the procedure described in the previous section. Figure 9a shows that the optimal starting frequency was 3.0 MHz, while figure 9b shows that the optimal ending frequency was 4.75 MHz. The dependence of the attenuation coefficient on the same starting and ending frequencies using the substitution method was also studied. It was found that the attenuation coefficient obtained from the substitution method was insensitive to small changes in these frequencies.

Fig. 7 Effect of changing window length on the bias error in the near field and far field. (a) with ABT. (b) without ABT. The error bars represent ±1 standard deviation. Percent overlap was 0%; bandwidth was 2.5 - 4.75 MHz. Error increases for window lengths greater than 1.4 cm.
Fig. 8 Effect of changing percent window overlap on the bias error in the near and far field. (a) with ABT, (b) without ABT. Error bars of 3-5% on all points are not shown. Error increases for percent window overlap >50%. Window length is 1.2 cm; bandwidth is 2.5-4.75 MHz.

Optimization of the signal processing parameters alone (without ABT), summarized in table 1, resulted in changes of the estimated attenuation coefficient slope from 0.30 to 0.35 dB cm\(^{-1}\) MHz\(^{-1}\) in the far field, which amounts to a reduction of the bias error from -29% to -17%. In the near field, the optimization resulted in a reduction of the bias error from -35% to -26%. These errors are still large. When ABT was added, the errors were further reduced. This decrease can be clearly seen from figures 10a and 10b. ABT further reduced the error of the optimized estimation in the far field from -17% to 0%, resulting in an unbiased estimate. The benefits of ABT in the near field were not as dramatic, amounting to a reduction of the bias error from -26% to -21%. It is interesting to note that the goodness of fit parameter \(r^2\) is maximized whenever the bias error is minimized, as seen in figures 11a,b.

Fig. 9 Effect of changes in passband on bias error in the far field. (a) error vs. starting frequency with and without ABT. (b) error vs. ending frequency with and without ABT. The optimal lower frequency is 3.0 MHz, while the optimal high frequency is 4.75 MHz. Window length is 1.2 cm; percent overlap is 46%. Error Bars of 5-8% on all points are not shown. For figure 9(b), error increases for ending frequencies greater than 4.8 MHz.
Table 1  Summary of results. Values of $\beta$ are means ± s.d.

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<th>FAR FIELD</th>
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<td>(1.2 cm, 46%, 3.0-4.75</td>
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<td>$0.33 \pm 0.03$ -21</td>
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<td>MHz) with ABT</td>
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IV. DISCUSSION AND CONCLUSION

The substitution measurements demonstrate a consistency between attenuation values measured in the near field and in the far field. Phase cancellation effects [19,20] did not give rise to discrepancies between values.

Fig. 10  Comparison of bias error due to diffraction vs. percent window overlap after optimization for window length and bandwidth. (a) far field. (b) near field. Note that the diffraction error is nearly eliminated in the far field at a window overlap of 46% when ABT is used. Error Bars of 3-5% on all points are not shown.
Fig. 11 The relationship between total bias error and the coefficient of determination. (a) total bias error. (b) coefficient of determination. Note that minima in the total bias error are accompanied by maxima in $r^2$.

obtained in the near field and in the far field of the transducer. This is probably due to the fact that the transducer used was unfocussed. It was found, however, that the results of the substitution method depend critically on the normal incidence adjustments, particularly in the far field. Great care had to be taken to properly make this adjustment.

It is evident that the spectral difference estimation technique is subject to bias errors, and that these errors tend to underestimate the actual attenuation value. We postulated that the total bias error could be written as

$$
e_{tb} = e_{wsb} + e_{wob} + e_{bwb} + e_{db}, \tag{2}$$

where $e_{tb} = $ total bias error,

$e_{wsb} = $ bias error due to window size,

$e_{wob} = $ bias error due to window overlap,

$e_{bwb} = $ bias error due to bandwidth, and

$e_{db} = $ bias error due to diffraction,

where the four bias terms on the right hand side of the equation are independent of each other. Thus, in general, we can write

$$
e_{db} = e_{tb} - e_{wsb} - e_{wob} - e_{bwb} \tag{3}$$

If we define

$$
e_{spb} = e_{wsb} + e_{wob} + e_{bwb}, \tag{4}$$

where $e_{spb} = $ bias error due to signal processing

then

$$
e_{db} = e_{tb} - e_{spb} \tag{5}$$
If we optimize the signal processing such that the value of $\varepsilon_{spb}$ approaches zero, then

$$\varepsilon_{db} = \varepsilon_{tb}.$$  

(6)

We then apply ABT which reduces $\varepsilon_{db}$ to near zero. Having done so, we get from Eq. (6)

$$\varepsilon_{tb} = 0.$$  

(7)

Table 1 summarizes the reduction in bias errors due to ABT in the near and far fields. The spectral difference estimation technique, when optimized for window length, window overlap and bandwidth, still produces a -17% bias error in the far field and a -26% error in the near field. Axial beam translation further reduces these bias errors to 0% and -21%, respectively. Thus, while the use of ABT produces unbiased estimates in the far field, it fails to substantially improve the estimation in the near field. This effect could be occurring since during data acquisition in the near field with ABT, the near field is occupied by varying thicknesses of the foam material, which could greatly influence the free formation of the beam. On the other hand, the varying thicknesses of foam in the far field occupy a relatively small part of the beam length, while the beam has already had the chance to form in the near field. In other words, ABT in the near field may progressively interfere with beam formation, and thus be ineffective in reducing the errors in the estimation, since the underlying assumption of constant beam properties is no longer valid.

The reduction of the signal processing bias was accomplished by observing the minima in bias errors due to its components. Window size would be expected to have an optimal range, since for large windows, fewer spectra will be obtained from a given thickness of phantom material, resulting in inferior quality of the linear regression, while small windows would give rise to fewer temporal samples which adversely affect the quality of the FFTs. The window overlap is a compromise between the correlation between windows and the number of windows. However, too great an overlap will result in correlated samples. Finally, a wider bandwidth will produce less error as long as the signal-to-noise ratio of the various spectral components remains high. At the edges of the passband, however, lower s/n ratios are encountered such that the error in the estimate is likely to increase as the bandwidth of the estimation increases beyond some point [22].

It can be seen from table 1 that the precision of the estimation is always inferior to the precision of the substitution measurement, as might be expected. However, the standard deviations of the estimations are still quite small when compared to those reported by Insana et al [16]. This is most likely due to the use of many more echo sequences, and the use of the double linear regression method.

In conclusion, we have demonstrated that it is possible to obtain unbiased estimations of attenuation in the far field of an unfocussed aperture using axial beam translation in tissue mimicking material.

The main disadvantage of the ABT technique is the need to move the transducer axially; methods to overcome this limitation are currently under investigation. On the other hand, unbiased estimates can be obtained without calibration, a process that is fraught with problems [18]. Axial beam translation may remove the error in the estimation due to tissue type and possibly also due to the effect of intervening tissues.
ACKNOWLEDGMENTS

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APPENDIX A

It has been shown [23] that the variance of the slope of the regression line $s^2$ is related to the variance of the residuals as

$$ s^2 = \frac{12s_r^2}{(dx)^2m^3} \quad (A1) $$

where

$dx =$ distance between adjacent data points along the abcissa,

$m =$ number of data points, and

$s_r^2 =$ variance of the residuals of the fit.

Hence the variance of the nth slope for the fits of amplitude vs. depth is given by

$$ s_n^2 = \frac{12s_{a,n}^2}{(DX)^2(NWINDOW)^3} \quad (A2) $$

where

$DX =$ window length,

$NWINDOW =$ number of windows, and

$s_{a,n}^2 =$ variance of the residuals of the nth fit.

These $n$ variances are averaged to give a mean variance $s_{avg}^2$. The variance of the fit to slopes of the previous fits vs. frequency is given by

$$ s_{B}^2 = \frac{12(s_{avg}^2 + s_b^2)}{(DF)^2M^3} \quad (A3) $$

where

$DF =$ frequency resolution,

$M =$ number of discrete frequencies in the passband, and

$s_b^2 =$ variance of the residuals of this fit.

REFERENCES


