Optimization of Speed-of-Sound Estimation from Noisy Ultrasonic Signals

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Abstract—The effects of prefiltering and the choice of time delay estimators and statistical data reduction techniques on the precision of speed-of-sound estimation were investigated using the beam-tracking technique. It was found that prefiltering the data with an ideal 50-kHz low-pass filter improved the precision of the estimation in all cases. Echo cross-correlation had an advantage over peak detection for low signal-to-noise ratio (SNR) levels, but its advantage diminished as the signal-to-noise level improved due to filtering. The linear regression method was superior to the paired-point analysis technique under all conditions. Using the optimal set of parameters, precision on the order of 0.1 percent was achieved in a tissue-mimicking phantom when one beam was tracked along 75 mm in 1-mm increments.

I. INTRODUCTION

PULSE-ECHO techniques have not been used for measuring the speed of sound in vivo until recently. Robinson et al. [14] described a discrete target method whereby they made use of the fact that, if an identifiable target within tissue is scanned from two directions, the resultant images of this target adequately overlap only if the speed-of-sound propagation in the tissue that is assumed by the scanner matches its actual value; the greater the mismatch, the less satisfactory will be the overlap. Bamber [2] has described an array scanner which, in conjunction with a biprism, generates two divergent images in the same target, as in Robinson’s techniques. Bamber reports error in accuracy on the order of 8 percent. An alternative technique for measuring the speed of sound in tissue between two transducers is the crossed-beam method [1]. [4]. Two transducers are arranged so that their axes of radiation intersect. The time of flight of the transmitted pulse to the intersection volume plus the time of flight of the backscattered echoes from there to the receiving transducer is computed. Assuming negligible effect of the speed of sound propagation in the body-wall fat layer, the total path length assumed from geometric considerations is divided by the total travel time to yield the speed of sound in tissue. Takakura et al. [5] and Hayashi et al. [3] have used the delay lines associated with focused arrays to adjust for the sharpest attainable ultrasonic image. From the delays so obtained, the speed of the target between the transducer and the target was calculated. Hayashi reports ±2 percent precision in liver in vivo. Ohnishi et al. [10] have reported a technique they call the “reference point technique,” which uses two reference points, such as the two edges of a tumor. The actual distance between the points is measured with an array transducer at right angles. They report an accuracy error in the measurement of about 10 percent. Ophir [12] has described a beam tracking technique that reports precision and accuracy errors of 1 percent or less in vitro, which cannot be directly compared with the previous in vivo results. The approaches discussed in this paper are designed primarily for this last technique but are applicable wherever measurements of arrival times of ultrasonic signals from intersecting beams are made.

A brief description of the beam tracking technique is presented to provide some background. The basic method is illustrated in Fig. 1. Two coplanar transducers are positioned such that their beam axes intersect at a known angle, typically π/2. This is an optimal setup that may be difficult to achieve in vivo. One of the transducers serves as a transmitter while the other acts as a receiver. A short pulse is first emitted from the transmitting transducer. Energy scattered from the volume of beam intersection $V_1$ arrives at the receiving transducer located at position $R_1$ at time $t_1$ after the emission of the pulse. The receiving transducer is then moved to a new location $R_2$, separated from the previous location $R_1$ by a small distance $\Delta x$. The new arrival time of the scattered energy from volume $V_2$ is now $t_2$, where $\Delta t_1 = t_2 - t_1$. The receiving transducer is then moved again to location $R_3$, and the quantity $\Delta t_2 = t_3 - t_2$ is again measured, and so on. The tracking interval $\Delta t$, which is under user control, depends on the characteristics of the transducer radiation field and the scattering properties of the target; a typical value might be $\Delta x = 1$ mm. After the receiving transducer has been positioned in $n$ locations and $n - 1$ values of $t$ are acquired (where typically $50 \leq n - 1 \leq 100$), a plot is made of $\Delta t$ versus $\Delta x$. A least squares linear regression fit is performed, and the slope of this fit is $c^{-1}$, where $c$ is the estimate of the speed of sound propagation along the path of the transmitted pulse. Variations in the speed of sound along this path are averaged out by the linear regression process. A more detailed discussion of the method is given in [12]. The trade-offs among the various scanning parameters and their effect on the precision of

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the estimation are discussed by Kontonassios and Ophir [7].

The precision of speed-of-sound estimation depends heavily on the ability to estimate time delays optimally. The problem of time delay estimation becomes a problem in statistics and signal processing, since the signal of interest is generally corrupted by a significant amount of noise. The presence of the noise increases the uncertainty with which the correct time delays can be determined.

In this paper we investigate the influence of statistical and signal processing considerations on the ability to estimate time delays, and hence the speed of sound in uniformly scattering targets using the beam tracking method. Specifically, we investigate the conditions under which the precision of the estimation is optimized. A flow chart describing our approach is shown in Fig. 2.

The signal-to-noise ratio (SNR) of the ultrasonic echo can be greatly increased by proper prefiltering (i.e., filtering prior to speed-of-sound estimation), since the signal and the noise occupy different passbands within the frequency spectrum. We use three filters in the analysis: 1) an all-pass filter, 2) a 100-kHz ideal low-pass filter, and 3) a 50-kHz ideal low-pass filter (LPF). Our results show that the use of the signals filtered with the 50-kHz LPF consistently produce the most precise speed-of-sound estimates.

Two general classes of time delay estimators are investigated. The first involves the difference between the temporal positions of the envelope peaks of signals acquired from two separate transducer locations. The second involves the computations of the cross-correlation function between two such signals, noting the delay that is required in order to maximize its value. We found that, in general, the cross correlation is a more precise estimator of time delay. This result is expected in light of the previous result, since the cross correlation itself is a low-pass filtering operation. However, we have also found that, given adequate prefiltering, the results obtained from the peak and the cross-correlation estimators are essentially identical.

We investigate two statistical methods for estimating the speed of sound. The first is the paired point method, whereby the estimated arrival times of two signals separated by a known distance are computed. The process is then repeated by sliding the pair of points along a tracking distance while maintaining their separation. The second method is the standard linear regression method, whereby delay estimates are fitted to a straight line using the least squares algorithm. We found that the linear regression method is considerably superior to the paired point method under all circumstances. A similar conclusion was reached previously by Wilson et al. [15] in their investigation of attenuation estimations.

The combined use of prefiltering, the correct estimator, and linear regression analysis produces a 95-percent confidence interval for the mean speed of sound that is on the order of ±0.1 percent of the mean, when a single beam is tracked in 1-mm increments along a 75-mm path in a tissue-mimicking phantom.

II. ANALYSIS

We discuss in the following the expected effects of prefiltering, choice of estimator, and statistics on the precision of the speed-of-sound estimator.

A. Effect of Prefiltering

Inspection of the typical demodulated ultrasound signal in Fig. 3(a) indicates the existence of a signal corrupted by high-frequency noise. This signal represents the cross-beam profile, since the transducers are arranged so that their beams cross. It is this signal whose arrival time we wish to estimate. Due to the high-frequency noise, low-pass prefiltering may be beneficial in improving the SNR of the estimation. Note that the width of the cross-beam profile is typically on the order of 20 μs in our experiments. This corresponds to an approximate passband of 50 kHz.

Fig. 4 shows the spectrum of the demodulated ultrasound signal shown in Fig. 3(a). Note that noise is present at frequencies well above 50 kHz. It is expected that prefiltering would be helpful in reducing the variance of the
estimation regardless of which algorithm (peak or cross correlation) is used. Two examples of the effects of pre-filtering of this signal are shown in Fig. 3(b), using low-pass filters at 100 kHz and 50 kHz. Note that the cross-beam profile is clearly present without the additive noise, especially with the 50-kHz filter.

B. Time-Delay Estimators

In general, the problem of estimating the speed of sound in tissues using reflected ultrasound involves the estimation of time delay between signals received by two (or more) spatially separated transducers in the presence of noise. The two signals can be mathematically described [6] as

\begin{align}
    x_1(t) &= s_1(t) + n_1(t) \\
    x_2(t) &= As_1(t + D) + n_2(t).
\end{align}

In the beam-tracking case, \( s_1(t) \) is related to the convolution of the tracked transducer impulse response with the tracking transducer cross-beam profile, which for a short impulse response is essentially equal to the beam profile. Even though the width of \( s_1 \) may change over a long track due to the changes in the radiation field of the transducers, these changes are typically symmetric and thus do not tend to influence the estimation of arrival times. Here \( s_1 \) is assumed to be shift invariant. \( A \) is a scaling factor, \( D \) is the time delay, and \( n_1(t) \) and \( n_2(t) \) are noise terms. Next we investigate the relative performance of peak and cross-correlation delay estimators and the effect of filtering on them.

1) Peak Signal Delay Estimator: The simplest form of delay estimation can be accomplished by looking at the difference between the time of arrival of the peaks of \( x_1(t) \) and \( x_2(t) \), i.e., the peak delay estimator is given as

\[ \hat{\theta}_{\text{peak}} = \text{TPP} \{ x_2(t) \} - \text{TPP} \{ x_1(t) \} \]

where the TPP operator denotes temporal peak position.

It is clear that the presence of the noise terms \( n_1(t) \) and \( n_2(t) \) would make this estimator a noisy one. Its advantage, on the other hand, is in its simplicity and computational speed. We have previously shown, using experiment [12] and simulation [7], that interference among echoes from within the area of beam overlap causes a stair-stepped appearance of this delay estimator, which increases the variance of the residuals of the linear regression fit to the time-delay estimators at sequential tracking transducer positions. This, in turn, proportionally increases the variance of the slope of the linear regression, which is an estimate of the inverse speed of sound.

Fig. 5(a) shows two demodulated echo signals acquired in a reticulated foam phantom [11] from positions separated by 3 mm. The main echo complex can be easily identified, and a high level of noise can be seen. It is clear that the peak method produces appreciable errors in the estimation from such noisy signals. Fig. 5(b) shows the same signals after low-pass filtering at 50 kHz. The peak positions clearly are more representative of the arrival times of the cross-beam profile.

2) Cross-correlation Delay Estimator: A common method for determining the time delay \( D \) is to compute the cross-correlation function [6]

\[ R_{x_1x_2}(\tau) = E[x_1(t) x_2(t - \tau)] \]

where \( E \) denotes expectation, and the argument \( \tau \) that
maximizes $R_{112}(\tau)$ is an estimate of the delay. Due to finite observation time, $R_{112}(\tau)$ must be estimated as

$$\hat{R}_{112}(\tau) = \frac{1}{T - \tau} \int_{\tau}^{T} x_1(t) x_2(t - \tau) \, dt$$

where $T$ represents a finite observation interval.

The time shift ($\tau$) causing a peak in the function $R_{112}(\tau)$ is an estimate $D_{\text{corr}}$ of the time delay $D$. Since the cross correlation is in itself a filtering operation, it would be expected that at least part of the noise seen on the signals in Fig. 5(a) will be eliminated when performing a cross correlation, which should result in improved precision of the delay estimation. This result is indeed borne out in the experiments.

C. Statistical Methods

We investigate the use of two statistical methods for the estimation of the speed of sound. The first is the paired-
point method, whereby the estimates of arrival times of two signals separated by a known distance is computed. The process is then repeated by sliding the pair of points along the total estimation distance while maintaining their separation. An average is taken of all such delay estimates. The second method is the standard linear regression method, whereby delay estimates are fitted to a straight line using the least squares algorithm. We develop equations for the precision error of both techniques and find the relationship between them.

1) Paired-Point Method: This method was described by Kuc and Schwartz [8] and was used for estimation of ultrasonic attenuation from pulse-echo data. In the present application, we try to estimate the difference of arrival times between pairs of noisy ultrasonic echoes. In general, the distance between observation points is \( \Delta x \), and we estimate transit times over windows of length \( l \Delta x \) up to the total observation length \( L \Delta x \). The quantities \( l \) and \( L \) are positive integers, and \( l \leq L \). If we assume a constant speed of sound \( c \) along the line of sound propagation, we would like to optimize the estimation of time delay \( \hat{\tau} \) due to a distance change of \( \Delta x \) by proper selection of \( l \) and \( L \). We may write the estimated time delay as

\[
\hat{\tau} = \bar{\tau} + t_{\alpha/2, n_p - 1} s_p / \sqrt{n_p}
\]

where

- \( \hat{\tau} \) : estimated time delay,
- \( \bar{\tau} \) : mean time delay,
- \( s_p^2 \) : variance of the measured time delays,
- \( t_{\alpha/2, n_p - 1} \) : the critical value of the \( t \) statistic leaving a fraction \( \alpha/2 \) of the probability in each tail, with \( n_p - 1 \) degrees of freedom.
- \( n_p = L - l + 1 \) : total number of pairs.

We define the fractional error relative to the mean for the paired point method as

\[
\epsilon_p = \left| \frac{t_{\alpha/2, n_p - 1} s_p / \sqrt{n_p}}{\bar{\tau}} \right|
\]

We wish to compute the ratio \( l / L \), which minimizes \( \epsilon_p \), under the assumption that \( s_p^2 \) remains constant. We rewrite (7) as

\[
\epsilon_p = \left| \frac{t_{\alpha/2, L - l} s_p / \sqrt{L - l + 1}}{c} \right|
\]

For \( (L - l) \geq 10 \), the value of \( t \) remains essentially constant. Since \( c, s_p^2, \) and \( \Delta x \) are also assumed constant, we write

\[
\epsilon_p = K \frac{1}{\sqrt{L - l + 1}}, \quad \text{where} \quad K = \left| \frac{t_{\alpha/2, L - l} s_p c}{\Delta x} \right|
\]

thus

\[
\epsilon_p = K f(l) \quad \text{where} \quad f(l) = l^{-1}(L - l + 1)^{-1/2}
\]

We now find the minimum of the function \( f(l) \) in terms of the quantity \( l / L \). Differentiating \( f(l) \) with respect and setting it to zero, yields

\[
\frac{d}{dl} f(l) = l^{-1}(-\frac{1}{2})(L - l + 1)^{-3/2} \left(\frac{-1}{L} \right) + (l^{-2})(L - l + 1)^{-1/2} \left(\frac{-1}{L} \right)
\]

\[
= l^{-1}(L - l + 1)^{-1/2} \left[ \frac{1}{2} (L - l + 1)^{-1} - l \right] = 0.
\]

The bracketed quantity has to equal zero, such that \( 3 \geq 2L + 2, \) or \( l \geq (2L + 2/3) \). Typically, \( 2L \gg 2 \), therefore the extremum is found at \( l = \frac{2}{3} L \). It can easily be shown that

\[
\frac{d^2}{dl^2} f(c) = l^{-1}(L - l + 1)^{-1/2}
\]

\[
\cdot \left\{ \left[ \frac{1}{2} (L - l + 1)^{-1} - l \right] \right\} > 0 \quad \text{(1)}
\]

if \( l^{-1}(L - l + 1)^{-1/2} > 0 \) or \( L \geq l \geq 1 \). This condition is always satisfied, and thus the existence of a minimum is confirmed. Therefore the optimal separation between the paired points that minimizes the value of \( \epsilon_p \) is

\[
\frac{l}{L} \approx \frac{2}{3} \quad \text{(10)}
\]

From (10) we change variables from \( l \) to \( l / L \), viz.

\[
f(l) = \frac{1}{l(L - l + 1)^{1/2}}
\]

\[
= \frac{1}{L \left( \frac{l}{L} \right) \left( 1 - \frac{l}{L} + \frac{1}{L} \right)^{1/2}}
\]

\[
= \frac{1}{L^{3/2} \left( \frac{l}{L} \right) \left( 1 - \frac{l}{L} + \frac{1}{L} \right)^{1/2}}
\]

\[
= L^{-3/2} \left( \frac{l}{L} \right) \left( 1 - \frac{l}{L} + \frac{1}{L} \right)^{-1/2}
\]

where \( g(l/L) \) is defined as the bracketed quantity, and

\[
\epsilon_p = KL^{-3/2} g\left( \frac{l}{L} \right) \quad \text{or} \quad \epsilon_p = KL^{-3/2} \left( \frac{l}{L} \right)^{-1/2}
\]

Fig. 6 shows the quantity \( g(l/L) \) from (14) plotted again.
The ratio between the errors $\epsilon_p / \epsilon_r$ can be easily computed for the case where $L/L = 2/3$ and $L \gg 1$. The following relationships are used:

$$n_p - 1 = L/3$$  \hspace{1cm} (21)

$$n_r - 1 = L - 1 = L$$  \hspace{1cm} (22)

$$\left[ 1 + \frac{1}{(1 - 1/L)L} \right]^{-1/2} = 1 - \frac{3}{2L}$$  \hspace{1cm} (23)

$$\left( 1 - \frac{1}{L^2} \right)^{1/2} = 1$$  \hspace{1cm} (24)

$$g(2/3) = 2.6$$  \hspace{1cm} (25)

$$n_p^{-1/2} = \left( \frac{L}{3} \right)^{1/2}.$$  \hspace{1cm} (26)

With these approximations it can be shown that

$$\frac{\epsilon_p}{\epsilon_r} = 0.75 \sqrt{L} \left( \frac{t_{\alpha/2,L/3}}{t_{\alpha/2,L}} \right) \left( 1 - \frac{3}{2L} \right) \left( \frac{s_p}{s_r} \right).$$  \hspace{1cm} (27)

For $L \gg 1$,

$$\frac{\epsilon_p}{\epsilon_r} \approx 0.75 \sqrt{L} \left( \frac{s_p}{s_r} \right).$$  \hspace{1cm} (28)

The linear regression method improved relative to the paired point as the number of points $L$ along the track increases.

III. EXPERIMENTAL MATERIALS AND METHODS

Experimental work was done using the apparatus shown in Fig. 7. A test phantom was constructed (details given later) and placed in a 60-gal temperature-controlled water bath. Two ultrasonic transducers (3.5 MHz, 13 mm, focused at 4–10 cm) are mounted on an $x$–$y$ precision positioning system such that their axes of radiation intersect in a plane at right angles. In a typical experiment the transmitting transducer position is held fixed while the receiving transducer position is incremented in 1-mm steps.

The electronic apparatus is controlled by a Compaq-286 personal computer via an IEEE Std. 488 bus system. The stepper motor controller (Superior Electric) enables the movement of either transducer in 2.5-μm increments. The transmitting transducer is shock-excited by the transmitter (Metrotek) at a rate of 1–2 kHz. The received signal is amplified by an input-protected custom preamplifier and fed into a computer-controlled attenuator and RF amplifier. The attenuator works in an interactive software-driven mode, so that the peak amplitude of the signal in a given time window of interest always occupies the input range required by the LeCroy digitizer to utilize its full 8-bit capabilities. In the present experiments the digitization rate was set at 12.5 MHz. All signal-processing operations are performed in software. Demodulation is performed on the received RF signal by computing its absolute value. Filtering was done on the demodulated signal in the frequency domain by first taking a 2048-point
fast Fourier transform (FFT) of a rectangular windowed signal centered around its peak. At the 12.5-MHz sampling rate, this window corresponds to about 160 μs. Frequency components above the desired filter cut-off frequency are eliminated, and an inverse transform was taken. All cross-correlation operations were done in the time domain using a 500-point window function centered around the peaks of each demodulated waveform.

In a typical experiment, the receiving transducer is positioned at some arbitrary initial position and the data are acquired. The receiving transducer is then repositioned in 1-mm steps, closer or farther from the transmitting transducer, and the process is repeated. Typically, 50-100 points are collected.

A 15.3-cm tall 7.16-cm diameter acrylic cylinder with 0.64-cm walls and one end closed served as the phantom container. A block of open-cell reticulated polyester foam (Scott Paper) was cut into a cylinder, which fits tightly in the container. The grade of foam used had an average of 20 pores per linear inch and contained open spaces in about 98 percent of its volume. Further details regarding the preparation of the phantom are given by Ophir et al. [11].

IV. RESULTS

A beam-tracking procedure was performed using the setup described above at 37.0 ± 0.5°C. Beam tracking was performed over a path of 75 mm in increments of 1.00 mm. Thus a data set of 75 waveforms was used in all calculations reported below. The effect of prefiltering on the demodulated waveforms has been shown qualitatively in Figs. 3-5.

A. Paired Point Method with Peak or Cross-Correlation

Fig. 8(a) shows the percent error in the estimate εp (as defined by (7)) as a function of the ratio (I/L), expressed in percent, for the peak estimator. The low-pass filter cut-off frequency is used as a parameter. We observe that the general behavior of all curves follows the theoretical behavior of Fig. 6. The reason for the craggy appearance of the experimental data is that the variance of the time delay estimator does not remain constant as the fraction (I/L) is changed. The theoretical derivation has assumed a constant variance, resulting in a smooth curve. It is evident that the magnitude of the overall error in the estimate diminishes as the cut-off frequency of the LPF is decreased, with approximately a five-fold improvement in going from the unfiltered case to the 50-kHz filtered case.

Fig. 8(b) shows the percent error εp as a function of the fraction (I/L) expressed in percent for the cross-correlation estimator. Again, the cut-off frequency of the low-pass filter is the parameter. While the general behavior of the curves remains similar, it is seen that the overall error level for the unfiltered signals is about half that of the unfiltered peak estimator case. However, filtering does not improve the estimation as efficiently as before. In fact, the 50-kHz filter produces an error curve that is almost identical to the 50-kHz peak estimator case. Evidently, with adequate filtering, enough noise is removed from the system such that the choice of estimator becomes relatively unimportant.

B. Linear Regression Method with Peak or Cross-Correlation

The results of the previous section were compared with those derived from linear regression analysis. Equations from previous sections were used to calculate the values of ε, and εp (calculated at the optimal I/L = 2/3). Table I shows the speed of sound estimates and the error performance of using both analyses under various prefiltering conditions. The ratios of the relative errors εp/ε, are also shown.

V. CONCLUSION

We have investigated the influence of prefiltering, and the choice of estimators and statistical methods on the precision of the speed-of-sound estimator. It was shown that the models used are realistic and that theoretically predicted results actually occur in experiments involving realistic data.

Prefiltering of the signals takes advantage of the fact that the raw signal, which is related to the crossbeam sensitivity profile, is corrupted by substantially higher frequency noise components. As a result, prefiltering with a low-pass filter improves the SNR of the signal, which ultimately results in improved precision of the speed-of-sound estimate [6]. This was seen to be a general result that is independent of the choice of estimator and/or statistical method.

The choice of estimator is important only as long as prefiltering is inadequate or totally absent. The cross-correlation estimator consistently outperformed the peak estimator under these conditions, as might have been expected. While the peak estimation algorithm is by nature very sensitive to noise spikes, the cross-correlation estimator is in effect a low-pass filter, which, as seen from
Fig. 8. Relative percent error of estimate, $e_p$, for paired-point method. Abcissa is in units of $(1/L)$ expressed in percent, where $L = 75$, $\Delta x = 1$ mm. (a) $e_p$ for peak estimator. (b) $e_p$ for cross-correlation estimator.

<table>
<thead>
<tr>
<th>Analysis Method</th>
<th>50 kHz</th>
<th>100 kHz</th>
<th>Unfiltered</th>
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<td>Regression</td>
<td>Peak x corr</td>
<td>Peak x corr</td>
<td>Peak x corr</td>
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<tr>
<td>Speed-of-sound estimate (m/s)</td>
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<tr>
<td>$e_p$ (percent)</td>
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<tr>
<td>Speed-of-sound estimate (m/s)</td>
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<tr>
<td>$e_p$ (percent)</td>
<td>6.765 6.917 9.157 8.061 11.549 7.351</td>
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</tr>
</tbody>
</table>

*The speed-of-sound estimate and the percent error are computed for the regression and paired-point analyses using various filtering schemes and choices of estimators.*

The choice of statistical method has a strong influence on the precision of the estimation. We have corroborated the theoretical results of Kuc and Schwartz [8] for the paired-point method, both theoretically and experimentally. However this method, even when optimized was shown experimentally to be substantially inferior to the linear regression method. This is an expected result since the previous paragraph, improves the signal-to-noise ratio. Nevertheless, when adequate prefiltering is applied, we observe that both peak and cross-correlation methods converge, i.e., there is no longer any particular advantage associated with the cross-correlation method. On the contrary, since the cross-correlation method is computationally intensive relative to the peak method, it becomes relatively unattractive under these conditions.
while the optimal paired-point procedure ignores the central one-third of the available data points, the linear regression method takes all of them into account [13]. A similar conclusion was reached by Wilson et al. [15] for attenuation estimation.

Inspection of Table I demonstrates the points made in the preceding. Regardless of statistical method or choice of estimator, prefiltering reduces the relative error in the estimation by several fold. Given a particular level of prefiltering, the cross-correlation estimator always gives results as good as (at 50 kHz) or better (at ≥ 100 kHz) than the peak method. The ratio between the relative error values of cross-correlation and peak is on the order of 0.5–1. By far, the greatest reduction in precision error is achieved when the linear regression is used, where εp/εr is as large as one order of magnitude. The absolute values of all prefiltered estimations are within a few meters per second.

In conclusion, the use of appropriate prefiltering in conjunction with linear regression allows the use of the computationally efficient peak estimator, resulting in actual precision errors, under the present experimental conditions, that are on the order of 0.1 percent.

REFERENCES


J. Ophir, photograph and biography not available at time of publication.

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