CORRELATION ARTIFACTS IN SPEED OF SOUND ESTIMATION IN SCATTERING MEDIA

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(Received 25 July 1988; in final form 24 November 1988)

Abstract—A recently described method for speed of sound estimation in tissues in pulse-echo mode involves reception of echoes generated by an ultrasonic pulse by means of a linearly tracking transducer. When the peaks of echo amplitudes are used as markers of arrival time, stairstep-like artifacts appear in the echo arrival time vs. transducer position plots. We postulate that these artifacts are a consequence of the speckle phenomenon commonly encountered in ultrasonic imaging. To test this hypothesis, we report computer simulations and water tank experiments which demonstrate similarities between the behavior of the stairsteps and the properties of ultrasonic speckle. Additionally, equations describing the precision of the speed of sound estimation in terms of the second order statistical properties of the stairstep artifact are derived.

Key Words: Speed of sound, Pulse echo, Beam Tracking, Speckle, Estimation, Correlation, Artifact, Scattering.

I. INTRODUCTION

In recent years a number of articles have been published which report various methods for speed of sound estimation in pulse-echo mode. The techniques proposed by Robinson et al. (1982) and by Bamber and Abbott (1985) rely on quantifying the displacement between two images of a discrete target obtained from two vantage points; the amount of such displacement is related to the error in the speed of sound assumption, and is quantifiable by cross-correlation techniques. A somewhat similar technique involving a discrete target is that due to Hayashi et al. (1985) and Katakur et al. (1985) whereby the sharpness of the target is maximized by interactive user control of signal delays at the aperture. Ohtsuki et al. (1985) has used discrete reflectors in conjunction with a linear array and a single element transducer to measure the speed of sound. Inuma et al. (1985) and Akamatsu et al. (1985) have used crossed beam methods to measure the total transit time of the pulse and the echo produced from diffuse scatterers. Ophir (1986) has reported a Beam Tracking technique whereby accurate and precise estimation of speed of sound in diffusely scattering phantoms and in tissues in vitro were demonstrated, and some of the signal processing and statistical considerations necessary for ameliorating the precision of the estimation were also discussed (Ophir et al. 1988; Kontonassios and Ophir 1987). A short description of the Beam Tracking approach is given in section II.

One of the observations made in using the Beam Tracking technique was that when the delays of peak echo amplitudes were plotted against the spatial translation of the tracking transducer, a stairstepped appearance always occurred (Fig. 1). This appearance was not sensitive to the types of target materials which were used, that is, reticulated foam blocks (Ophir et al. 1981), fine particles in a gel matrix (Madsen et al. 1978), and normal human liver samples (Ophir and Kontonassios 1987). In a one dimensional computer simulation of the Beam Tracking process in randomly scattering media, similar stair-step artifacts occurred as well (Kontonassios and Ophir 1987). These are termed artifacts since plateaus in the data imply a corresponding infinite sound speed, which is clearly unrealistic.

The existence of the artifacts causes deterioration in the quality of the speed of sound estimation, which is based on fitting a linear function to the delay vs. position data. Specifically, an increase in the average size of the stairsteps causes an increase in the variance of the residuals of the fit, which reduces the precision of the estimation. Additionally, the ultimate resolution of the Beam Tracking process is compromised due to bias errors which increase when
Typical Beam Track

Peak Arrival Times

![Graph showing arrival times vs. distance along track.](image)

Fig. 1. Typical experimental stairstep data obtained by tracking over 30 mm and recording the arrival time of the peak of the echo complex.

the ratio of total track length to average stairstep length is reduced. Thus understanding this artifact and dealing effectively with it is of prime importance.

In this paper we investigate the origin of the stairstep artifact using computer simulations and water tank experiments. We test the postulate that this artifact is essentially related to the speckle phenomenon in ultrasound imaging (Burckhardt 1978; Wagner et al. 1983). Hence the average stairstep size, defined in terms of the autocorrelation length of the residuals of the linear fit to the data, is shown to critically depend on the size of the tracking aperture. Moreover, no sensitivity to the density of scatterers in the insonified region is demonstrated as long as this density is sufficiently high. We also investigate the consequences of this artifact in terms of the limitations which it imposes on the quality of the speed of sound estimation obtainable, and suggest methods for minimizing these limitations.

II. BEAM TRACKING METHOD

We give below a short background description of the principles of the Beam Tracking method and the variables which affect its precision (Ophir 1986; Kontonassios and Ophir 1987).

A two-dimensional model of the Beam Tracking method operating in the body is illustrated in Fig. 2. Two coplanar transducers are positioned such that their beam axes intersect at right angles. The transducer whose beam is tracked serves as a transmitter, and the other whose beam performs the tracking serves as a receiver. Initially, the receiving transducer is positioned at some arbitrary position $R_1$. The energy scattered in the volume of intersection of the tracked and the tracking beams $V_1$ resembles a noisy Gaussian modulated carrier waveform which is unsuitable for direct time delay measurement. Upon full wave demodulation and low pass filtering, the envelope of the signal is produced and the time $t_1$ required for the pulse to travel from the transducer to $V_1$ and then to $R_1$ is measured as the arrival time of the peak echo.

The receiving transducer is then moved to a new location $R_2$, separated from the previous location $R_1$ by a distance of $\Delta x$. The new arrival time of the peak echo from volume $V_2$ is now $t_2$. The receiving transducer is moved again by $\Delta x$ and the time $t_3$ is again measured, and so on. The chosen tracking increment $\Delta x$ depends on the characteristics of the transducer radiation field and the scattering properties of the target. After the tracking transducer has been positioned in $i$ locations and $i$ values of $t$ are acquired, a plot is made of $t$ vs. $x$. A least squares linear regression fit is performed, and the slope of this fit is $\hat{\beta} = 1/\hat{c}$, where $\hat{c}$ is the estimate of the speed of sound propagation along the path of the tracked beam.

It can be shown (Kontonassios and Ophir 1987) that the precision of the estimate $\hat{\beta}$ is given by

$$\text{Precision}_m(\hat{\beta}) = \pm 2\left(\frac{3s^2\Delta x}{ml^3}\right)^{1/2}, \quad (1)$$

where $s^2$ is the estimated variance of the residuals, $l$ is the total length of the track, and $m$ is the number of uncorrelated tracks. This expression shows that the precision of the inverse speed of sound estimator may be improved by increasing the number of tracked beams, increasing the length of each tracked beam, decreasing the sample spacing, and/or decreasing the variance of the residuals. Although the first and second factors above improve the precision of the estimator, they require an increase in scanned area, which is ultimately limited to the size of the tissue or region of interest. Decreasing the increment $\Delta x$ improves the precision, but only as long as the decorrelation of the samples can be maintained. The occurrence of the stairsteps could be thought of as a manifestation of the lack of such decorrelation over a finite tracking length; the physical reasons and techniques for dealing with this problem are the main topics addressed in this paper. Finally, reducing the variance of the residuals is a desirable way to improve the precision (Kontonassios and Ophir 1987). This variance can be reduced by proper choice of estimators or by appropriate filtering (Kontonassios and Ophir 1987; Ophir et al. 1988).
III. ANALYSIS

The stochastic nature of the stairsteps does not allow for meaningful direct measurement of their sizes. Instead we look at their second order statistical correlation properties. We first remove the linear trend in the stairsteps. This is done by fitting a straight line to the data using the least squares criterion, and subtracting the resulting trend from the respective stairstep data. The resultant random process \( r(x) \) depicts the residuals of the fit. We then compute the discrete autocorrelation function of the random process \( r(x) \), viz.

\[
R_r(\Delta x) = \sum_i r(x_i)r(x_i + \Delta x),
\]

where \( \Delta x \) is the shift along the \( x \)-axis. A convenient form of the autocorrelation function is obtained by normalizing with respect to the autocorrelation function obtained at zero shift, viz.

\[
\hat{R}_r(\Delta x) = \frac{R_r(\Delta x)}{R_r(0)}.
\]

The magnitude of \( \hat{R}_r(\Delta x) \) has unity magnitude at zero shift. We define the value of \( \Delta x \) at which \( \hat{R}_r(\Delta x) \) first reaches a value of 0.5 as \( L \), the correlation length of the stairsteps. Figures 3a–c show an example of the various steps in this procedure.

We postulate that the stairstep artifact is related to the speckle phenomenon in ultrasound imaging (Burckhardt 1978; Wagner et al. 1983). Specifically, the artifactual appearance of the data is a result of coherent interference (speckle) among echoes emanating from a multiplicity of small scatterers and which are detected by a finite receiving aperture. In the limiting case when the target material consists of many randomly dispersed fine particles (i.e., much smaller than the wavelength), the resulting fully developed speckle carries information only about the point spread function of the system and none about the target (Wagner et al. 1983).

If the stairstep artifact is indeed a manifestation of the speckle phenomenon encountered in ultrasonic imaging, it would be expected to behave in a similar manner to the behavior of speckle. This would imply that (a) the correlation length of the stairsteps would be insensitive to the density of the scatterers as long as many small scatterers are present in the insonified region, and that (b) the correlation length of the stairsteps would be directly related to the autocorrelation properties of the aperture in the transverse plane.

The insensitivity of correlation length of the stairsteps \( L \) to the scatterer density will be demonstrated via computer simulations and via experiments reported later in this paper. The autocorrelation properties of an aperture in the transverse plane were elucidated by Wagner et al. (1983) and by Burckhardt (1978). Their analyses rely on a continuous wave formulation which forms the basis for first-order estimate of the point spread function of the transducer. In the Fraunhofer field of a circular piston transducer operating in Beam Tracking mode (where the directivity of the tracked transducer can be ignored) the point-spread function is given by

\[
h(x) = B J_1(\pi \xi x)/(\pi \xi x),
\]

where

\[
\xi = D/\lambda z;
\]

\( D = \) aperture diameter,
\( z \) = the axial distance from the aperture to the insonified region,

\( B \) = normalization factor, and

\( J_1(\cdot)/(\cdot) = j_{\text{inc}} \) function.

In the Fraunhofer field of a rectangular aperture we have

\[
    h(x) = A \sin(\pi x)/(\pi x) = A \text{sinc}(\pi x),
\]

(5)

where now \( D \) refers to the transducer dimension in the tracking direction. This simpler form can now be used for circular transducers provided we replace \( D \) by \( D' = D/1.08 \), that is, replacing the diameter of the circular aperture with a smaller quantity \( D' \) (Wagner et al. 1983). \( A \) is a normalization factor.

The transverse autocorrelation of the aperture \( R_h(\Delta x) \) can be computed by using the convolution relationships, since the function \( h(\Delta x) \) is even (Brigham 1974). Therefore

\[
    R_h(\Delta x) = Ah(\Delta x) \ast Ah(\Delta x) = A \text{sinc}(\pi \Delta x) \ast A \text{sinc}(\pi \Delta x),
\]

(6)

where \( \ast \) denotes correlation, and \( \ast \) denotes convolution. This convolution can easily be carried out in the Fourier domain. The Fourier transform of \( A \text{sinc}(\pi \Delta x) \) is a gate function \((A/\xi)G(\omega)\) of height \( A/\xi \) and width \( \xi \). The convolution above is a multiplication of the two functions in the Fourier domain which yields \((A/\xi)^2G(\omega)\), that is, a gate function of height \((A/\xi)^2\) and width \( \xi \). Taking the inverse transform yields the desired autocorrelation function of the aperture

\[
    R_h(\Delta x) = (A^2/\xi) \text{sinc}(\pi \Delta x).
\]

(7)

The width of the main lobe of this autocorrelation function is \( 2/\xi = (2\lambda/D)z \), that is, the width of the autocorrelation function of the aperture in the Fraunhofer field is proportional to the distance \( z \) of the insonified region from the tracking aperture, and is inversely proportional to the aperture diameter \( D \).

We make use of this relationship later in our experimental setup, where the aperture autocorrelation width at a fixed range in the Fraunhofer field is increased by employing circular acoustic baffles in front of the transducer aperture.

The transmitted pulse envelope shape along the tracked beam was assumed to be a Dirac function, and therefore it had no effect on the determination of \( R_h(\Delta x) \). In practice, the pulse envelope shape along the range direction resembles a Gaussian (Wagner et al. 1983)


\[ p(t) = (2\pi \sigma_x^2)^{1/2}\exp\left(-t^2/2\sigma_x^2\right), \quad (8) \]

where \( \sigma_x \) is defined as the pulselength. For a typical wideband medical transducer operating at 5 MHz in tissue, this quantity translates to a distance which is on the order of a wavelength (\( \sim 0.3 \text{ mm} \)). Thus this length is about an order of magnitude smaller than the beamwidth at the focus of a typical transducer, and therefore the Dirac shape assumption is maintained.

### IV. SIMULATION

A two dimensional simulation of the beam tracking process was performed on a computer, in order to test the dependence of the stairstep correlation length \( L \) on scatter density and on aperture dimensions. The two dimensions are in the plane of the beam tracking measurement.

#### a. Scattering model

The medium in which ultrasonic propagation occurs was assumed to consist of a highly populated two dimensional array of identical randomly distributed point scatterers whose mean density could be changed. All scatterers were assumed to have a fixed scattering cross-section, and first order scattering was assumed. It was further assumed that the medium of propagation was lossless and that the speed of sound in it was a constant 1500 m s\(^{-1}\).

#### b. Transducer model

Both tracked and tracking beams were assumed to have constant beamwidths throughout their radiation fields. The transverse diffraction profiles of the transducers were assumed to be Gaussians whose variances \( \sigma_x^2 \) and \( \sigma_y^2 \) could be changed, and thereby the effective size of the insonified region could be varied.

The impulse response \( h(t) \) of the tracked (transmitting) transducer was a sine wave modulated by a Gaussian envelope function, viz.

\[ h(t) = \exp\left[-t^2/2\sigma_x^2\right]\sin[2\pi f_0 t], \quad (9) \]

where

\[ \sigma_x^2 = \text{the variance of the envelope, and} \]
\[ f_0 = \text{the center frequency of the transducer}. \]

For the \( i \)th scatterer, the total transit time \( \tau_i \) required for this pulse to propagate from the center of the tracked aperture to the scatterer and on to the tracking aperture was computed from trigonometric considerations and by assuming the speed of sound given above. Thus the delayed version of this pulse is

\[ h(t - \tau_i) = h(t)\delta(t - \tau_i) = \exp[-(t - \tau_i)^2/2\sigma_x^2]\sin[2\pi f_0(t - \tau_i)], \quad (10) \]

where \( \delta \) is a Dirac function, and \( \tau_i \) is the \( i \)th time delay. This delayed pulse is convolved with the impulse response of the tracked transducer \( h(t) \) to result in

\[ g(t - \tau_i) = h(t)\star h(t - \tau_i). \quad (11) \]

The process is repeated for all \( N \) scatterers contained in the area of beam intersection. The scattering strength of the \( i \)th scatterer is computed as

\[ A_i = \exp\left[-\frac{(x_i - x_0)^2}{2\sigma_x^2} - \frac{(y_i - y_0)^2}{2\sigma_y^2}\right], \quad (12) \]

where \( x_0 \) and \( y_0 \) are central coordinates of the radiation axes of the two transducers, \( x_i \) and \( y_i \) are the rectangular coordinates of the \( i \)th scatterer, and \( 2\sigma_x \) and \( 2\sigma_y \) are the beamwidths of the tracking and tracked transducers, respectively, in the region of beam intersection.

The total waveform \( y(t) \) received by the tracking transducer is computed from a summation of \( N \) individual waveforms, that is,

\[ y(t) = \sum_{i=1}^{N} A_i g(t - \tau_i). \quad (13) \]

#### c. Procedure

Several simulations were carried out in order to investigate the dependence of \( L \) on (a) the density of scatterers, (b) the autocorrelation properties of the scatterers, (c) the beamwidth, and (d) the pulselength.

1. **Dependence of \( L \) on scatterer density.** This dependence was investigated by selecting two dimensional scatterer densities in the range between 0.75 and 3.0 mm\(^{-2}\). The beamwidths of the transducers in the region of intersection were both 2.41 mm, as defined by their values of \( 2\sigma_x \) and \( 2\sigma_y \). For each density, Beam Tracking was performed with the following parameters:

   - center frequency = 3.0 MHz
   - bandwidth = 2.0 MHz
   - tracking increment (\( \Delta x \)) = 1.0 mm
   - tracking length = 75 mm.

   The value of \( L \) was computed for each scatterer density.

2. **Behavior of \( L \) with "zero beamwidth" transducer.** These simulations were done to show that if a new uncorrelated constellation of scatterers is used
for every new position of the tracking transducer, the value of $L$ would be essentially zero, that is, the function $\tilde{R}_c(\Delta x)$ is a Dirac function. This situation is not a realistic case, but rather a limiting case which would occur when the transducer beamwidth became vanishingly small.

Two simulations were performed. In the first one, the constellation of scatterers remained fixed in space, and a beam tracking procedure was implemented with the following parameters:

- center frequency = 3.0 MHz,
- bandwidth = 2.0 MHz,
- scatterer density = 75 mm$^{-2}$,
- tracking increment ($\Delta x$) = 0.2 mm,
- beamwidth ($2\sigma_x$) = 2.41 mm.

In the second experiment, all parameters remained unchanged, but a new random constellation of scatterers was computed each time the tracking beam was incremented by $\Delta x$, thus assuring complete decorrelation of signals derived from adjacent positions of the tracking beam, even though $2\sigma_x \gg \Delta x$. This procedure is necessitated by the fact that as $\sigma_x$ approaches 0, the number of scatterers within the width of the beam diminishes such that no statistically meaningful results are obtainable.

3. Dependence of $L$ on the beamwidth. This dependence was investigated by performing Beam Tracking simulations with different values of beamwidth of the tracking transducer. The parameters used in the simulation were as follows:

- center frequency = 3.0 MHz,
- bandwidth = 2.0 MHz,
- tracking increment ($\Delta x$) = 0.5 mm,
- ranges from tracking transducer = 5 cm,
- beamwidths of tracking transducer $2\sigma_x$ = 1.43, 1.92, 2.41, 3.38, 4.36 mm,
- beamwidth of tracked transducer $2\sigma_x$ = 2.41 mm.

4. Dependence of $L$ on the pulsewidth. The (temporal) excitation pulsewidth $2\sigma_t$ was varied from 0.5–2.0 $\mu$s in several steps, for a beamwidth of 2.41 mm and a center frequency of 5 MHz, and the corresponding values of $L$ were calculated. The process was repeated 5 times, and mean and $sd$ values of $L$ were computed.

d. Results

Results of the simulations are shown in Figs. 4 through 10. The dependence of $L$ on scatterer density was shown to be weak for densities greater than 0.5 mm$^{-2}$ (Fig. 4). When the scatterer constellation was randomly changed whenever a tracking increment occurred, autocorrelation of the residuals becomes essentially impulsive, as expected (Fig. 5a, b). Beamwidth is clearly shown to affect the value of $L$ (Figs. 6–8) and the variance of the residuals (Fig. 9), while pulsewidth is shown to have no discernable effect on $L$ (Fig. 10).

V. EXPERIMENT

The computer simulations have shown that the autocorrelation lengths of the residuals $L$ were related to the beamwidth and independent of the density of scatterers. We conducted Beam Tracking experiments in a water tank in order to corroborate the simulation results.

a. Procedure

Beam tracking experiments were performed on four tissue mimicking reticulated polyester foam sponges with different porosities (Ophir et al. 1981) in a water tank at $37 \pm 0.5^\circ$C. Prior to their immersion in the water tank, the sponges were submerged in a beaker containing distilled water and degassed in a desiccator using laboratory vacuum pressure of 500 mbar for 20 minutes.

The transducer whose beam was tracked was a 13 mm aperture KB-Aerotech Quik-Scan #65034H, 3.5 MHz focussed transducer used in transmit mode. The $-6$ dB beam radius of the shock excited transducer was measured using a 3 mm diameter stainless steel ball target. The beam radius is seen to stay relatively constant at 2.5 mm at ranges between 50–80 mm. This range was used to perform the Beam Tracking experiments (Fig. 11).

The tracking transducer was an Echo #E87X81, 3.25 MHz 21 mm aperture unfocussed transducer used in receive mode. All tracks were performed in the far field at a range of 200 mm from the tracking transducer. The beamwidth of this transducer at this
fixed range was adjusted by using one of a set of four rubber baffles. The baffles were constructed from a 9.5 mm thick rubber absorber material containing a circular window. Window diameters were 8.9, 12.7, 15.8, and 19.1 mm. Additionally, the un baffled (21 mm) aperture was used for a total of five effective apertures. Baffles were affixed to the transducer aperture by using a small amount of silicone grease, and pressure was applied to squeeze out air bubbles which might be trapped between the baffle and the aperture. The attenuation in an intact piece of the baffle materia rial was separately determined by substitution to be >100 dB at the center frequency of the transducer. Thus the baffles provided a convenient means for constricting the effective aperture of the tracking transducer. The theoretical and experimental -6 dB widths of the main lobe of the transducer radiation pattern resulting from monochromatic insonification in the far field as a function of aperture radius are shown in Fig. 12.

Experimental work was done using the apparatus shown in Fig. 13. The ultrasonic transducers are
RF amplifier. The attenuator works in an interactive software driven mode, so that the peak amplitude of the signal in a given time window of interest always occupies the input range required by the LeCroy digitizer in order to utilize its full 8 bit capabilities. In the present experiments the digitization rate was set at 25 MHz. All signal processing operations are performed in software. Demodulation is performed on the received RF signal by computing its absolute value.

In a typical experiment, the receiving transducer beam was positioned 50 mm from the transmitting transducer and the data were acquired. The receiving transducer was then repositioned in 0.5 mm steps.

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**Fig. 6.** Simulation data with tracking aperture of 1.43 mm. (a) Data with linear fit. Observe small stairsteps. (b) Autocorrelation of residuals.

**Fig. 7.** Simulation data similar to Fig. 6 but with tracking aperture of 4.36 mm. (a) Data with linear fit. Observe large stairsteps. (b) Autocorrelation of residuals.
Correlation Length vs Beam Width
With Tracked Beam Width of 1.43 mm

Fig. 8. The correlation length $L$ vs. tracking beam width. Data are mean and $sd$ of the correlation length over 4 tracks for each beamwidth. An ascending trend in the data is evident.

Correlation Length vs Pulse Width
Beam Width is 2.41 mm

Fig. 10. Effect of pulsewidth on the correlation length $L$. No definite trend is detectable.

away from the transmitting transducer, and the process was repeated. Sixty data points were collected per track. The foam target was then translated lateral to the scan plane by a distance $\geq 5$ mm, and Beam Tracking was repeated. Three to four tracks were performed with each baffle. Peak echo amplitude arrival times were plotted against relative tracking transducer positions. Linear regression fits to the data were performed and the trend in the data removed. The autocorrelation functions of the remaining residuals were calculated, and the $-6$ dB widths of the autocorrelation function were calculated and averaged for all beam tracks involving a given baffle size.

Each foam was characterized by measuring the number of pores per linear cm in several locations. The average of the measurements was cubed, and used to estimate the number of pores per cm$^3$. This number was multiplied by the volume of the region of beam intersection to yield to total number of pores in this region. The number of pores may be considered as a relative measure of the number of actual scatterers in the region of interest.

b. Results

The mean $-6$ dB autocorrelation lengths of the residuals are shown in Fig. 14 as a function of measured beam radius, along with the standard deviation. The trend in the data demonstrates an increase of the autocorrelation length as a function of increase in beamwidth, which is similar to the simulation results shown in Fig. 8. The dependence of the mean residual correlation length on scatterer density is shown in Fig. 15. No significant dependence was found, corroborating the simulation results of Fig. 4.
Beam Radius vs Axial Distance
Tracked Transducer

Fig. 11. Experimental mapping of the -6 dB radius of the tracked transducer beam. The region between 50–80 mm range has a uniform minimal beam radius.

Fig. 12. Theoretical and experimental -6 dB beamwidth vs. aperture radius in the Fraunhofer field (200 mm) of the tracking transducer. Experimental values were obtained by using circular acoustic baffles in front of the transducer aperture.

Fig. 13. Experimental apparatus.
VI. DISCUSSION AND CONCLUSION

We have tested the hypothesis that the stairstep artifact in speed of sound estimations using the Beam Tracking technique is related to the speckle phenomenon commonly encountered in ultrasonic imaging. The test criteria used were (a) the lack of sensitivity of the correlation length of the residuals to the scatterer density at high densities, and (b) the dependence of the correlation length on tracking transducer beamwidth.

We have shown that the behavior of the correlation length of the stairsteps $L$ satisfies both criteria. The simulation results confirm that for scatterer density which exceeds about 0.5 mm$^{-2}$, the value of $L$ fluctuates very little. Similarly, the experimental work shows that markedly different scattering targets produce similar values of $L$. The dependence of $L$ on beamwidth was also demonstrated in simulation and experiment. Hence we conclude that the stairstep artifact satisfies both criteria set forth above and therefore appears to be related to speckle.

The stairstep artifact represents the ultimate resolution limitation of the Beam Tracking instrumentation. Clearly, this artifact causes distortion of the actual speed of sound estimation. The bias in the estimation introduced due to the occurrence of the stairstep artifact depends on the size of the stairsteps relative to the total length of the track, viz.

$$\text{bias error } \hat{\beta} \sim \frac{L}{l},$$

where $l$ is the overall length of the track. While it is always possible to increase $l$ and thereby reduce the bias error, for a given $l$ the bias error can be reduced only by reducing $L$. The results of this work demonstrate that this can in turn be achieved by ensuring that the estimation is performed in regions of small $L$, that is, in the focus of a highly focussed tracking aperture.

In general, the precision error in Beam Tracking in the absence of the stairstep artifact is governed by eqn (1),

$$\text{precision}_m(\hat{\beta}) = \pm 2 \left[ \frac{3s^2\Delta x}{ml^3} \right]^{1/2}.$$  \hspace{1cm} (1)

The presence of stairsteps adds to $s^2$ a new variance $s_L^2(L)$ which is approximately proportional to the

Fig. 14. Experimental correlation length vs. beam radius. A dependence similar to that of the simulation result (Fig. 8) is shown.

Fig. 15. Experimental correlation length vs. foam porosity. No discernible trend is seen. Note similarity to the results obtained from simulation (Fig. 4).
square of the correlation length of the stairsteps as shown in the appendix, *viz.*

\[ s^2(L) \approx k_1L^2, \]  

(15)

where \( k_1 \) is a positive constant of proportionality; moreover, since eqn (1) is only valid for uncorrelated measurements, the tracking increment \( \Delta x \) must be increased to a new value \( \Delta x_1 \) in a manner proportional to \( L \) in order for eqn (1) to remain valid, thus

\[ \Delta x_1 = \Delta x + kL, \]  

(16)

where \( k \) is a positive constant of proportionality. The precision becomes

\[ \text{precision}_m(\hat{\beta}) \simeq \pm \frac{3(s^2 + k_1L^2)(\Delta x + kL)}{mL^3}^{1/2}. \]  

(17)

When \( L = 0 \) (i.e., \( \hat{R}_e(\Delta x) \) is a Dirac function) the stairsteps disappear (Fig. 5a, b) and eqn (17) reduces to eqn (1).

Actual computation of precision using eqn (17) is not trivial. The variance of the residuals \( s^2 \) is dependent on the properties of the scattering medium. We have shown (Ophir et al. 1988) that prefiltering of the signal prior to the computation of peak amplitude, as well as the use of other estimators such as centroid or cross correlation lag are capable of reducing the precision error in the speed of sound estimation. The use of these techniques amounts to modifying the constants \( k \) and \( k_1 \) in eqn (17) such that the numerator is reduced. However, it was observed that in general the correlation artifact cannot be completely eliminated. It appears that a combination of highly focused transducers and optimal filtering is indicated for best performance; since the characteristics of the filter will depend on beamwidth, these two parameters must optimally balance each other. In a recent paper (Ophir et al. 1988) it was shown that under certain reasonable conditions it is possible to reduce the precision of the estimation of speed of sound in tissue mimicking phantoms to about 0.1%.

It is noteworthy that the stairstep artifact is not necessarily limited to the Beam Tracking method. Since we have shown that it is closely related to the speckle phenomenon encountered in pulse-echo ultrasonic imaging, it would be expected to be present in various guises in any pulse-echo sound speed estimation technique which relies on diffuse echoes which are dominated by speckle. While the phenomenon may not always take on the stairstep appearance, the dependence of the precision of the estimation on speckle in such techniques would be expected to be a fundamental limitation.

Acknowledgement—This work was supported by NIH Grant R01-CA44389.

REFERENCES


APPENDIX—RESIDUAL VARIANCE VERSUS STAIRSTEP LENGTH

We compute the dependence of the variance of the residuals on the length of the stairstep. We make a simplifying assumption
that all stairsteps are of uniform size (the average size) and that they have a sawtooth shape. Under these assumptions we can write the expression for the variance in some special cases shown in Fig. 16(a–c).

1. Sawtooth length along the abscissa = 2Δx. In this case we have \( j = 3 \) sample points. The variance of the residuals is

\[
s_j^2 = \frac{1}{3} \cdot \frac{2(1^2)}{3} = 1.
\]

2. Sawtooth length = 4Δx. We have \( j = 5 \) sample points, and the variance of the residuals is

\[
s_j^2 = \frac{1}{5} \cdot \frac{2(1^2 + 2^2)}{5} = 2.5.
\]

3. Sawtooth length = 6Δx. We have \( j = 7 \) sample points, and the variance of the residuals is

\[
s_j^2 = \frac{1}{7} \cdot \frac{2(1^2 + 2^2 + 3^2)}{7} = 4.67, \quad \text{and so on.}
\]

Generalizing, we write

\[
s_j^2 = \frac{2}{j - 1} \sum_{i=1}^{j-1/2} i^2. \tag{A1}
\]

The well known (Beyer 1985) sum of the series of squared integers

\[
\sum_{i=1}^{j} i^2 = \frac{j(j + 1)(2j + 1)}{6}, \quad \text{and thus}
\]

\[
\sum_{i=1}^{(j-1)/2} i^2 = \frac{[(j - 1)/2][(j - 1)/2 + 1][j]}{24} = \frac{j^3 - j}{24}. \tag{A2}
\]

This result is valid for odd and even values of \( j \). The general expression for the variance of the residuals is derived by combining eqns (A1) and (A2), viz.,

\[
s_j^2 = \frac{2}{j - 1} \cdot \frac{j^3 - j}{24}. \tag{A3}
\]

For values of \( j \gg 1 \), indicating small \( \Delta x \) (i.e., the stairsteps are represented by many sample points), we have

\[
s_j^2 \approx \frac{1}{12} j^2. \tag{A4}
\]

Since for a fixed value of \( \Delta x \) the quantity \( j \) represents the length of the stairstep, it is clear that the variance of this simplified depiction of the residuals is parabolically dependent on the length of the stairsteps. Assuming that the correlation length \( L \) is proportional to the average length of the stairsteps, it follows that

\[
s_j^2 \approx k_1 L^2, \tag{A5}
\]

where \( k_1 \) is a constant of proportionality. This equation is repeated as eqn (15) in the text. Figure 9 derived from the simulation shows a dependence between the variance of the residuals and the beamwidth which is suggestive of a parabola.

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Fig. 16. Model for calculation of residual variance. Distance in units of \( \Delta x \), and time in units of \( \Delta t \). (a) Small residuals. (b) Intermediate residuals. (c) Large residuals.