REDUCTION OF IMAGE NOISE IN ELASTOGRAPHY

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Elastography is a method for imaging the elastic properties of compliant tissues which produces gray scale elasticity images called elastograms. The elastograms of phantoms with homogeneous elastic properties exhibit a noisy appearance. We demonstrate that this noisy appearance of the elastograms is due to the nonstationary relationship between the pre- and postcompression signals that results in an artifactual modulation of the strain estimates by the amplitude variations of the envelope of the rf signal. We have identified two methods to reduce the strain modulation artifact. The first method consists of reducing the signal amplitude swings within the observation windows by logarithmically or otherwise compressing the rf signal. The sensitivity of this method to amplitude compression strength and the ability to reduce the noise in the elastograms without affecting the spatial resolution are investigated through simulations. The second method to reduce the strain modulation artifact consists of temporal stretching of the signal obtained after physical compression to approximate the shape of the signal obtained before compression. In this paper, we discuss the first method. The results show that significant improvement in image noise can be obtained with logarithmic amplitude compression. This improvement is obtained in conjunction with improved spatial resolution.

Key words: Crosscorrelation; elasticity; elastogram; elastography; imaging; logarithmic compression; noise; strain; strain modulation; stretch; spatial resolution; ultrasound.

INTRODUCTION

Elastography, a method for imaging the elastic modulus of compliant tissues, has been introduced recently [1-5]. The method estimates local strain directly from the estimates of one dimensional changes in small tissue elements. Tissue displacement estimates are obtained using crosscorrelation analysis of A-lines obtained pre- and post-target compression. Methods for crosscorrelation analysis of time shift for tissue motion and elasticity assessment using ultrasound have been described by many authors in the literature, and are summarized in [1]. Local strain measurements can be combined with theoretically and experimentally derived information about the stress distribution in the target to obtain a quantitative elastic modulus.

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image, or elastogram. In a simpler formulation of elastography, the stress distribution is not taken into account (assumed constant), and strain images (elastograms) are generated. Ponnekanti et al [3] have shown that a constant stress is a reasonable approximation when large compressors are used. The second approach is used in this paper.

In general, elastograms exhibit good spatial and parametric resolution. We have shown experimentally that a 5 dB difference in elastic modulus can be readily detected in a phantom composed of layers with different elasticities, and a spatial resolution on the order of a millimeter is achievable [1]. We have also shown that small hard inclusions on the order of 3 mm in diameter embedded in an elastic medium can be easily detected [2]. In a companion paper [4], we show that elastography is feasible in vivo.

The elastograms of phantoms with homogeneous elastic properties exhibit a noisy appearance. Since this noisy appearance is not directly related to ultrasonic speckle in conventional B-scans, we refer to it as elastographic noise. At least two factors contribute to the noisy appearance of elastograms. First, while the elasticity of the synthetic foam phantoms we have used is globally homogeneous, some real local variability is expected. Second, errors in the time shift correlation estimations result in a temporal uncertainty that causes random noise in the local strain estimates, and ultimately noise in the elastogram. A main component of this noise is due to the degraded crosscorrelation of pre- and post-target compression A-line segments that are deformed with respect to each other. We discuss the mechanisms by which the signal deformation mediates the undesired modulation of the time shift measurements.

In this paper, we discuss a method for reducing this effect that involves nonlinear compression of the rf signal amplitude. We demonstrate that through this method a substantial improvement in image quality is possible without loss of spatial resolution. A second method for reducing the strain modulation artifact is to temporally stretch the post-target compression signal in order to approximate the shape of the signal before target compression. We present a preliminary evaluation of this method; the full evaluation of stretching in elastography is currently under investigation.

STRAIN MODULATION ARTIFACT

Estimates of local strain along a line of sight are obtained by measuring the displacement (or time shift) between pre- and post-target compression A-line segments, obtained from observation windows that in general overlap in time. The time shift is estimated using the normalized crosscorrelation function [7]. Consecutive observation window pairs are taken with a uniform spacing ($\Delta T$), that in general is smaller than the window size (10% to 20% of the window size). After target compression, the spacing between consecutive window pairs is reduced by the amount

$$[\Delta t(i) - \Delta t(i-1)],$$

where $\Delta t(i)$ is the time shift between windows in the indexed window pair (Fig. 1). The local longitudinal strain at the depth given by the product $i \times (\Delta T \times c \times 2)$, where $c$ is the assumed
speed of sound in the elastic medium, is calculated as

\[ s(i) = \frac{\Delta t(i) - \Delta t(i-1)}{\Delta T}. \]  \hspace{1cm} (1)

The compression of the tissue causes a reduction of the scatterer spacing that results in distortion of the post-target compression A-line. A theoretical analysis describing the decorrelation of A-lines before and after target compression has been developed by Wear et al [8]. Due to the deformation of the rf signal after target compression, we must consider the following concepts:

a) Interwindow compression. The reduction of the spacing between consecutive window pairs before and after target compression contains the elasticity information we wish to extract. The local strain is estimated from this information via Eq. (1).
Fig. 2  A typical window pair with 1% compression. As a result of the compression, the window pair can only match in one region of the window. Window pairs shifted to match at the beginning of the window (a), at the center of the window (b), and at the end of the window (c) are shown.

b) Intrawindow compression. Due to the small magnitude of the applied target compression (usually less than 1%), the signal from the compressed tissue is distorted such that it resembles a time scaled version of the pre-target compression signal. The major complication due to this distortion is that, although the pre- and the post-target compression signals are individually stationary, they are jointly nonstationary [6]. The nonstationarity of the signals impairs the time shift estimation using crosscorrelation; a unique match of the window pair is not possible and signals can match exclusively in certain areas of the window. This is shown in figure 2 where signals from a typical window pair are shown matching either at the beginning (Fig. 2a), at the center (Fig. 2b), or at the end (Fig. 2c). The best match occurs when the shift between signals is such that the mean square error between them is minimized. This minimum mean square error shift is given by the location of the peak of the crosscorrelation function of the signals (see Appendix A). In an ideal case where there are no statistical fluctuations of the signal envelope within the window, the minimum mean square match occurs when the center of the pre- and post-target compression windows are aligned. However, for the practical case where we have random envelope fluctuations within the window, the mean square error match deviates from the center in favor of regions of large envelope amplitude within the window. Hence, the estimate of the time shift between two signals obtained before and after target compression is affected by the signal amplitude. Since the strain estimates are obtained directly from the time shift measurement by Eq. (1), this effect results in an undesired random modulation of the strain estimates by the random signal envelope amplitude. The distribution of signal envelope amplitudes follows some statistical distribution (e.g., Rayleigh if there is a large number of small randomly distributed scatterers), and the added variability due to the strain modulation effect is related to that envelope amplitude distribution. Since in general the signal envelope amplitude is independent of the elasticity of the medium, the elastographic noise introduced by the undesirable leakage of signal envelope amplitude information into the strain estimations is considered an artifact.

We have identified two methods for reducing the strain modulation artifact. The first method is based on the reduction of the signal envelope amplitude fluctuations by compressing the rf amplitude. This is done through a nonlinear transformation of the signal according to some rule, such as logarithmic compression or soft limiting. In this way, amplitude swings of the rf can be controlled by the strength of the applied log compression prior to the crosscorrelation operation. The second method applies a temporal stretch of the signal from
the compressed target in order to approximate the shape of the signal before compression of the target. Although a temporal stretch is not the inverse transformation of the signal deformation due to physical target compression, we believe it may improve the estimation since it will at least align the rf signal peaks. Note that this method requires a priori knowledge of the local target compression. The use of time companding correlators is well known in applications where time shift estimation is needed in presence of Doppler time scaling [9,10]. This paper is devoted to the first method.2

SIMULATION

We have tested these algorithms on simulated data generated by a computer program. The advantage of working with simulated data is that we can model the elasticity of the target as being perfectly uniform; any variability observed in an elastogram obtained from simulated rf is related only to the degree of the artifact, and does not represent true elasticity fluctuations in the target.

The simulation program of elastography experiments consists of two main parts: a model for the ultrasound transducer, and a model for the compliant scattering medium. Simulated A-lines are constructed by adding up the impulse responses of the transducer at the locations of all scatterers in a defined region of interest, considering appropriate travel times and beam spreading effects. The location of the scatterers is recomputed based on the simulated compression and the new A-line is constructed. The A-lines are simulated using floating point arithmetic.

A full description of a fixed aperture transducer model and of the scattering medium was presented in an earlier paper [11]. In brief, the transducer is modeled as a two dimensional sampled aperture composed of point subtransducers; the spacing between subtransducers is \(\lambda/2\), where \(\lambda\) is the wavelength in the target associated with the center frequency of the transducer. Each subtransducer is considered as a point source or receiver having a two-way Gaussian transfer function. The impulse response of the whole aperture at a given point in the radiation field is computed as the summation of appropriately delayed impulse responses of the subtransducers. The scattering medium is represented by a two-dimensional array of random numbers corresponding to the coordinates of the individual point scatterers. Multiple scattering is ignored.

The compliance of the simulated phantom is introduced by evaluating the relocation of the scatterers due to mechanical compression. The relocation of the scatterers depends on the stress distribution within the simulated phantom. Based on a three dimensional spatial model, the distribution of longitudinal stress (and thus strain) can be computed based on the boundary conditions and the structure of the target using commercially available finite element analysis software (Pal2, MacNeal-Schwendler Corporation, Los Angeles, CA). For the purposes of this paper, we have chosen to use a simpler one-dimensional model in which the target is compressed between two large compressing surfaces. The compressor applies unidirectional stress to the target such that all scatterers within the ultrasonic beam move in the direction of the applied compression. The applied stress is assumed to propagate uniformly so that the localized stress is constant throughout the medium. The simulation

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2A brief evaluation of the second method is included for completeness following the suggestions of one of the reviewers. The second method will be the subject of an independent paper.
Fig. 3 Equivalent spring model for a columnar tissue element as used in the simulation program. Springs represent segments of the tissue column with different Young's moduli.

The program models a target consisting of a background of uniform elasticity that contains a horizontal layer of uniform but different elasticity. The displacement of each scatterer is a function of the applied strain and the position of the scatterers within the layered model; the displacement of each scatterer is modeled by considering an equivalent one-dimensional spring system where the springs represent columnar tissue elements of cross-sectional area A and the scatterer of interest is located at the node between springs (Fig. 3). The spring constants are a function of the Young's moduli of the tissue elements and of their lengths. Since this model considers a static compression, viscous and inertial terms are not considered.

The equation governing the displacement $d$ of a scatterer due to a displacement $\Delta y$ is

$$d = \Delta y \times \left[ \frac{\sum_{i=1}^{M} l_i}{Y_i} \right] + \left[ \frac{\sum_{i=1}^{N} l_i}{Y_i} \right],$$

where $l_i$ is the length of the $i$th layer, $Y_i$ is the Young's Modulus of the $i$th layer, $N$ is the total number of columnar tissue elements in the equivalent spring system, and $M$ is the number of columnar tissue elements between the scatterer and the plane of compression. This equation is derived in Appendix B.

The simulation program constructs the precompression A-line from a number of randomly located scatterers. The location of each scatterer is then changed according to Eq. (2) and a post-target compression A-line is constructed. The process is repeated for a number of different locations in the simulated phantom to obtain many independent A-line pairs.
METHODS

Logarithmic amplitude compression was applied to the digitized rf signal according to

\[ y \log(n) = \text{sign}[y(n)] \times \log_{10}[1 + \text{CS} \times \text{abs}[y(n)]] \]  

(3)

where \( y(n) \) is the digitized rf signal and \( \text{CS} \) (compression strength) is a factor which determines the magnitude of the amplitude compression. The logarithmic amplitude compression was applied in software to the rf data which is assumed to be in the -2.5 V to 2.5 V range. We consider one bit quantization as the limiting case of log compression. One bit quantization has been previously proposed by Bonnefois and Pesque for blood flow and tissue motion measurements [12,13] and has been evaluated for 2D correlations [14]. The signals were converted to 1 bit signals using the following transformation:

- if \( y(n) \geq 0 \), then \( y(n) = 1.0 \)
- if \( y(n) < 0 \), then \( y(n) = -1.0 \).  

(4)

A temporal stretch of the post-target compression signal was applied using a linear interpolation algorithm [15]. Although linear interpolation is generally not appropriate [15], the interpolation accuracy improves with increased oversampling. In this preliminary study, we have chosen to oversample and to use the linear interpolation.

We designed a series of simulations to study the improvement of elastogram noise with the above processing methods and to evaluate any consequential loss of resolution. In all simulations, the crosscorrelation processing was performed using 6.5 ms overlapping windows, with 0.65 \( \mu \)s spacing between consecutive windows. Elastographic image quality was measured by the mean-to-standard deviation ratio (MSDR):

\[ \text{MSDR} = \mu_{S}/\sigma_{S} \]  

(5)

where \( \mu_{S} \) and \( \sigma_{S} \) are respectively the mean and standard deviation of the strain in a region of uniform elasticity.

A. MSDR simulation (uniform target)

Log compression strength studies were performed on a set of 30 uncorrelated A-line pairs obtained from simulation of a scattering target with uniform elasticity and a transducer with center frequency 5.0 MHz, 50% bandwidth, and 20 mm diameter, which was focused at 120 mm. The sampling rate was 100 MHz. Logarithmic compression strengths ranging from \( \text{CS} = 10^{-2} \) to \( \text{CS} = 10^{8} \) were tested. The simulation was repeated with 0.5 \%, 1.0 \%, and 1.5 \% strain levels. The MSDR was computed in each case from the entire set of strain values contained in the elastogram.

B. Resolution evaluation (layered target)

A simulated 20 A-lines of a 30 mm deep target containing a 10 mm horizontal layer in the center was used. The elastic modulus of the layer was set to 6 dB below that of the background (6 dB softer). The transducer specifications are the same as above. The applied target strain was 0.5 \%. The data were processed with and without log compression of the signal amplitude, with one bit transformation, and with temporal stretching. In order to
observe any loss of resolution of the edges between layers, average strain profiles were computed for the theoretical model of the phantom, the original elastogram, and the elastogram computed with log compressed rf, one bit transformation, and 0.5% linear temporal stretching. The distance from the 0.5% strain point to the 0.9% strain point measured on the rising edge of the average strain profiles was chosen as a nonrigorous measure of the axial spatial resolution.

RESULTS

Figure 4 shows a typical plot of the MSDR of the elastogram as a function of the log amplitude compression strength CS ranging from $10^{-2}$ to $10^8$. The MSDR of the elastograms obtained from the original rf (without log compression) and the one bit transformation are presented in this plot as limiting situations.

In figure 5, we show elastograms of the theoretical model for the layered target, the original elastogram, and the elastograms derived from log compressed rf, one bit transformation, and 0.5% linear temporal stretching. The average values of the MSDR of the elastograms in regions inside and outside the layer are given in table 1.

Figure 6 shows the corresponding average strain profiles which describe the ability of the image to depict the layers of the target.

DISCUSSION

The layered target simulation shows an improvement in the MSDR of the elastogram when the rf signal amplitude is log-compressed, one bit transformed, or when the signal is stretched in time (table 1). The softer middle layer can be observed in the original elastogram, but it is better defined in the images obtained with any of the other algorithms. It is apparent from figures 5c,d that improved spatial resolution can be achieved using the log compressed rf

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Fig. 4 Typical mean MSDR of the elastogram vs. signal amplitude compression strength obtained from a simulated tissue phantom of uniform elasticity for 0.5%, 1.0%, and 1.5% strain. The standard deviation of the MSDR estimates is less than 4%. The MSDR of the original elastogram and the one bit case are included as limiting situations. In general, the absolute MSDR values are a function of the log compression strength as well as the strain, the window size, the step size, and the frequency and bandwidth of the transducer.
Fig. 5 Elastograms of a 30 mm target that contains a 10 mm horizontal layer with a 6 dB difference in elasticity (simulation). A 0.5 % strain was applied to the target. The MSDR values inside and outside the layer are given in Table I. (a) Ideal elastogram of the computer model, MSDR = ∞; (b) Elastogram obtained from the original rf; (c) Elastogram obtained with amplitude log compression of the rf; (d) Elastogram derived with one bit transformation of the rf; (e) Elastogram derived with 0.5% linear temporal stretching of the postcompression signal.

or the 1 bit rf, as evidenced by the sharper transition between layers. A quantitative measure of the axial resolution (Table I) indicates an approximate threefold improvement when log compression is used. This is obtained in conjunction with a twofold improvement in MSDR. When temporal stretching is used, a significant loss of spatial resolution (relative to that achievable with log compression) can be observed as indicated by the widening of the transition zone between layers. The explanation of the resolution trade offs is beyond the scope of this paper.

Table I. Mean MSDR values obtained from inside and outside the layer for the elastograms in figure 4. The standard deviation of the MSDR values is less than 4%. A measure for the axial resolution is also shown.

<table>
<thead>
<tr>
<th></th>
<th>original rf</th>
<th>log compression</th>
<th>one bit</th>
<th>0.5% temporal stretch</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSDR inside layer</td>
<td>2.1</td>
<td>4.1</td>
<td>3.3</td>
<td>3.9</td>
</tr>
<tr>
<td>MSDR outside layer</td>
<td>1.8</td>
<td>4.5</td>
<td>3.6</td>
<td>7.9</td>
</tr>
<tr>
<td>0.5% to 0.9% strain distance [mm]</td>
<td>3.7</td>
<td>1.4</td>
<td>1.0</td>
<td>4.3</td>
</tr>
</tbody>
</table>
Fig. 6 Average strain profiles for the elastograms in figure 5 as follows: (a) Elastogram obtained from the original rf; (b) Elastogram obtained with amplitude log compression of the rf; (c) Elastogram derived with one bit transformation of the rf; (d) Elastogram derived with 0.5% linear temporal stretching of the post compression signal.

An example of the sensitivity of the MSDR of the elastogram vs. amplitude log compression strength is presented in figure 4. The MSDR plot presents a maximum: in this case the maximum occurs for CS=100. The MSDR is lowest without log compression, increasing rapidly for log compression strength values up to CS=100, and then decreasing slowly for larger log-compression strengths. The one bit transformation yields a MSDR that is inferior to the log compression case. We can observe in figure 4 that the MSDR improves with reduced target strain. This is a result of the reduced distortion of the post-target compression signal with respect to the pre-target compression signal with reduced strain. However, the strain cannot be reduced arbitrarily since we need to introduce tissue displacements that are above the threshold of sensitivity of the time shift estimator. Note that figure 4 presents typical curves and in general the absolute value of the MSDR in the elastogram is a function of the signal bandwidth, the noise in the signal, the center frequency, the strain, the step size, and the window size. The individual effect of these parameters on the MSDR and their interaction is the subject of a separate study. Preliminary studies suggest that improved MSDRs can be obtained for longer windows and bigger step sizes, at the expected expense of reduced spatial resolution in the images.

The elastogram obtained using temporal stretching exhibits the highest MSDR, particularly outside the soft layer. It is important to note that in this simulation we had a priori information about the strain (we applied 0.5% strain) and thus we were able to apply a temporal stretch which was nearly optimal for the area outside the layer. The improvement of the MSDR inside the soft layer is smaller since the temporal stretch is inappropriate. In
practice, the strain in the target is unknown, and therefore a global temporal stretch would not suffice. An iterative local stretching algorithm that adaptively stretches the signals until an optimum is obtained is under investigation.

CONCLUSION AND SUMMARY

The simulations presented in this paper demonstrate the presence of an artifactual variability in elastograms of uniformly elastic media (elastographic noise). This effect is evidenced by a relatively low MSDR in the images, which can be increased by compression of the rf amplitude or by temporal stretching. Logarithmic amplitude compression achieves the desired reduction of the variability of the rf envelope, which reduces the uncertainty in the time shift estimation. The results indicate that there is an optimal strength of the log compression, and that the sensitivity of the MSDR is low for compression strengths beyond this optimum. We postulate that initially the amplitude compression efficiently reduced the effect of the strain modulation artifact resulting in a rapid initial increase of the MSDR. However, as we increase the strength of the compression, the sampling rate gradually becomes less appropriate for the representation of the higher spectral components of the log compressed signal, resulting in a slow deterioration of the time shift estimates (and thus the MSDR). The one bit transformation was presented as a limiting case of the log compression and resulted in an improved, but less than optimal, MSDR. However, this MSDR may be improved further with increased sampling rates. The potential for fast hardware implementation of the one bit correlator makes this alternative especially attractive.

The improvement in the MSDR using logarithmic amplitude compression is achieved without sacrifice in resolution. In fact, a significant improvement of the axial resolution is apparent whenever log compression or one bit transformation is used. The resolution worsens when temporal stretching is applied. Resolution studies that include the investigation of these phenomena are underway. It should be noted that the techniques presented here are signal preprocessing techniques. Image postprocessing techniques may achieve similar reduction of the MSDR, but at the expense of reduced spatial resolution.

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APPENDIX A

Equivalence of the crosscorrelation function peak and the minimum of the mean-square-error function [16].

Given two discrete time zero mean signals u(n) and v(n) of length N, the crosscorrelation function Ruv is given by

\[ Ruv(k) = \sum_{n=1}^{N} u(n) \times v(k-n), \]  

(A1)
and the mean square error function \( \varepsilon \) is given by

\[
\varepsilon(k) = \sum_{n=1}^{N} [u(n) - v(k-n)]^2
\]  
(A2)

We can expand the square of the difference in Eq. (A2) to obtain

\[
\varepsilon(k) = \sum_{n=1}^{N} [u(n)]^2 + \sum_{n=1}^{N} [v(n)]^2 - 2 \times \sum_{n=1}^{N} u(n) \times v(k-n),
\]  
(A3)

\[
\varepsilon(k) = \sum_{n=1}^{N} [u(n)]^2 + \sum_{n=1}^{N} [v(n)]^2 - 2 \times Ruv(k).
\]  
(A4)

Since the first two terms in Eq. (A4) are independent of the index \( k \), the minimum of \( \varepsilon(k) \) corresponds to the maximum of \( Ruv(k) \).

APPENDIX B

Given a one dimensional system of \( N \) compressible tissue elements of elastic constants \( K_i \) under a dimensional compression \( \Delta y \), the sum of the dimensional compressions \( \delta_i \) of the individual elements must satisfy

\[
\Delta y = \sum_{i=1}^{N} \delta_i.
\]  
(B1)

Since the system is one dimensional and static, the stress in all element is equal. Therefore, for any two elements \( i, j \),

\[
\delta_i \times K_i / A = \delta_j \times K_j / A
\]  
(B2)

with

\[
K = Y / l \times A,
\]  
(B3)

where \( A \) is the area of the tissue elements, and \( Y \) and \( l \) are the Young's Modulus and the length of the subscripted element, respectively.

The displacement with respect to the plane of applied compression of a point located between elements \( i \) and \( i+1 \) is

\[
d_i = \sum_{k=1}^{i} \delta_k.
\]  
(B4)

The dimensional compressions of the \( i^{th} \) element \( \delta_i \) can be expressed in terms of the dimensional compression of the \( j^{th} \) element \( \delta_j \) and the Young's moduli and length of those
elements using Eqs. (B2) and (B3),

$$\delta_j = \delta_i \times (Y_j \times l_i / Y_i \times l_j).$$  \hfill (B5)

Using Eq. (B5) to express the dimensional compression of all N elements in terms of the compression of the \(j^{th}\) element (and their lengths and Young's moduli and lengths), Eq. (B1) can be rewritten as

$$\Delta y = \delta_j + \sum_{i=1}^{N} \frac{Y_j \times l_i}{Y_i \times l_j}$$  \hfill (B6)

and extracting common factors,

$$\Delta y = \delta_j \times (Y_j + l_j) \times \left( \sum_{k=1}^{N} \frac{l_k}{Y_k} \right).$$  \hfill (B7)

Thus, the compression in the \(j^{th}\) element can be expressed in terms of the applied compression and the length and Young's Modulus of all the tissue elements

$$\delta_j = \Delta y \times (l_j + Y_j) \times \left( \sum_{k=1}^{N} \frac{l_k}{Y_k} \right),$$  \hfill (B8)

and therefore using Eq. (B4) the displacement of a point located between elements \(i\) and \(i+1\) is

$$d_i = \sum_{k=1}^{i} \delta_k = \Delta y \times \left[ \sum_{k=1}^{i} \frac{l_k}{Y_k} \right] \div \left[ \sum_{k=1}^{N} \frac{l_k}{Y_k} \right].$$

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