ULTRASONIC IMAGING OF THE STRESS DISTRIBUTION IN ELASTIC MEDIA DUE TO AN EXTERNAL COMPRESSOR

H. PONNEKANTI, J. OPHIR and I. CESPEDES
Ultrasonics Laboratory, Department of Radiology, The University of Texas Medical School, Houston, TX 77030, USA

(Received 16 March 1993; in final form 26 July 1993)

Abstract—We describe an experimental ultrasonic method capable of imaging the two-dimensional distribution of longitudinal stress in an elastic, tissue-like material due to an external compressor of arbitrary size or shape and boundary conditions. The method involves the use of a compressor and an opposing ultrasonic transducer. Local strains are derived from the ultrasonic backscatter signals before and after compression using cross correlation analysis. The strain distribution is converted to a stress map by assuming a linear stress–strain relationship. The technique is useful for quantifying the corrections that must be made to images of the elastic modulus of tissue (elastograms) due to the effects of compressor size and shape, depth and boundary conditions. It is also useful for experimental modeling of stress distributions in elastic media.

Key Words: Elasticity, Elastic modulus, Elastogram, Elastography, Stress, Strain, Ultrasound.

INTRODUCTION

Recently we have described elastography as a method for imaging the elastic modulus distributions in tissues and tissue-like materials using ultrasonic waves (Ophir et al. 1991). Elastography experiments carried out on uniform tissue-mimicking phantoms have shown that the estimated elastic modulus in an elastically uniform target suffers an artificial depth dependent increase; we termed this artifact "target hardening." The artifact causes general darkening of the image of the inverse elastic modulus (elastogram) in deep regions and is dependent on the external source(s) of compression. This effect is due to the existence of a nonuniform distribution of stress in the target due to the geometry of the external source(s) of compression; it is evident in uncorrected elastograms (Ophir et al. 1991).

The distribution of stress in elastic materials has been investigated by several authors. The theoretical problem of determining the stress field in a semi-infinite elastic medium due to a compressor of known size and shape pressed against the free surface is known as the Boussinesq problem (Boussinesq 1885). Exact solutions for the stress distribution under circular compressors have been presented by Sneddon and Goodier (1970) and by Terazawa (1916). Chu and Li (1980) have described simple analytic models for the distribution of stress under a circular compressor using two different boundary conditions: when the compressor is assumed to be uniformly loaded, and when the compressor is assumed to have been subjected to a uniform displacement. Their models were corroborated by using a photoelastic stress-freezing technique. Closed form solutions were obtained for the stress produced in a semi-infinite solid by uniformly distributed pressure on part of the boundary by Love (1929). Saada (1974) has described analytically the stress distribution under circular and rectangular compressors. Influence charts have been described by Newmark (1947), which give stress or displacement information due to a given load geometry. Ultrasonic speed of sound tomography has been used for nondestructive evaluation of residual stresses in materials (Finlayson 1983; Begin 1981; Hildebrand and Harrington 1981). Experimental and analytical studies of the longitudinal stress distributions for circular compressors on a three-dimensional tissue model, were made by Ophir et al. (1991) and by Ponnekanti et al. (1992).

The correction required to compensate for the target hardening artifact in elastography is a function of the size and shape of the compressor, the depth of interest, the distance from the axis of the compressor, and the boundary conditions. A simple one-dimensional illustration of the target hardening effect and the necessary axial correction has been given by Ophir
et al. (1991). In cases where the ultrasonic beam axis does not coincide with the compression axis, the target hardening artifact, and therefore the required correction to overcome it, is complicated. Specifically, the artifact would be different for every ultrasonic beam produced by a transducer array of the type routinely used in diagnostic sonography, because such beams are translated with respect to the compression axis. For this reason it is important to know the two-dimensional behavior of the artifact. The correction of the elastogram for the two-dimensional target hardening effect is performed by multiplying the strain value of each pixel by the reciprocal of the number of the corresponding element of a normalized array of correction factors. The normalized reference array is a two-dimensional representation of the longitudinal stress distribution in a uniform, elastic, isotropic target that is being subjected to exactly the same boundary conditions. The maximum value of stress in the array is normalized to unity. Therefore, the corrected elastograms have units of inverse Young's modulus (kPa⁻¹).

In this paper, we describe a special procedure to generate images of the distribution of longitudinal stress in an elastic target under a compressor of known geometry. The images are corroborated using known analytical models. In general, this experimental procedure can be used to investigate the longitudinal stress field of compressors of arbitrary shape and size. The elastograms so obtained also allow proper correction for the target hardening artifact encountered in elastography.

**THEORY**

The general theoretical expression for the three-dimensional distribution of stress in a semi-infinite elastic solid under a uniformly stressed rectangular compressor was given by Love (1929). His expressions for the distribution of the vertical component of stress are

\[
\sigma_z(x, y, z) = \frac{1}{2\pi} \left[ \partial V / \partial z - z \partial^2 V / \partial z^2 \right],
\]

where,

\[
\partial V / \partial z = -p \Omega,
\]

\[
\Omega = (2 \pi - \alpha - \beta),
\]

\[
\alpha = \cos^{-1} \left[ \frac{(a - x)(b + y)}{\sqrt{(a - x)^2 + z^2}} \sqrt{(b + y)^2 + z^2} \right]
\]

\[
\beta = \cos^{-1} \left[ \frac{(a + x)(b - y)}{\sqrt{(a + x)^2 + z^2}} \sqrt{(b - y)^2 + z^2} \right]
\]

and

\[
\partial^2 V / \partial z^2 = p \left\{ \frac{a - x}{(a - x)^2 + z^2} \left[ \frac{b - y + b + y}{a_1} \right] + \frac{a + x}{(a - x)^2 + z^2} \left[ \frac{b - y + b + y}{b_2} \right] + \frac{b - y}{(b - y)^2 + z^2} \left[ \frac{a - x + a + x}{a_1} \right] + \frac{b + y}{(b + y)^2 + z^2} \left[ \frac{a - x + a + x}{d_4} \right] \right\},
\]

where \(\sigma_z\) is the stress component in the vertical direction (parallel to the z-axis) at a point \((x, y, z)\), \(p\) is the uniformly distributed applied pressure and \(V\) is the Newtonian potential. A compressive pressure is considered positive. The compressor is of length \(2a\) and width \(2b\), and \(a_1\), \(b_2\), \(c_3\) and \(d_4\) are the distances of the point \((x, y, z)\) from the compressor corners A, B, C and D, respectively (see Fig. 1).

Matrices of magnitude values of \(\sigma_z\) for all the points in the plane of interest where \(y = 0\), were generated and displayed in the form of two-dimensional gray scale images for two cases; case 1 when \(a = 40\) mm and \(b = 30\) mm (Fig. 2a), and case 2 when \(a = 30\) mm and \(b = 40\) mm (Fig. 3a). The brighter regions in the images indicate higher stress magnitudes.

**MATERIALS AND METHODS**

All experiments were performed in a 120 gal water tank at room temperature. A submerged, reticulated, 30 pores per inch block of polyester foam 200 mm \(\times\) 140 mm \(\times\) 140 mm was used as the acoustic/elastic target. The foam block was first degassed in a vacuum chamber for approximately twenty min using laboratory vacuum. Several drops of surfactant (Bath-Kleer, Instrumentation laboratories, Lexington, MA) were added to reduce the surface tension and thus eliminate air bubbles more efficiently.

The method employs a rectangular plexiglas compressor attached to an ultrasonic transducer (transducer
1; single element, 13 mm diameter, 3.5 MHz and focused at 4–10 cm) such that the axis of the compressor coincides with the axis of the transducer and an identical opposing transducer mounted flush into a plexiglas table (Fig. 4). Transducer 1 was used only for the initial alignment and the data were acquired using

![Diagram](image)

**Fig. 1.** Compressor geometry showing the various parameters.

(a)  
(b)  
(c)

**Fig. 2.** Panel of images for case 1: length = 80 mm, width = 60 mm. (a) Image of stress distribution—analytical data; (b) image of stress distribution—experimental data; and (c) image of the normalized difference between (a) and (b).
transducer 2. The foam block is interposed between the compressor and the transducer. The compressor is attached to a computer controlled precision three-dimensional translation device, which is capable of moving the compressor in a plane that contains the axis of the ultrasonic beam of transducer 2.

The initial alignment of the compressor axis with the axis of transducer 2 was done by pulsing transducer 1 and receiving with transducer 2 (see Fig. 4). The compressor was moved around in a plane parallel to the table (XY plane) in steps of 1 mm until a maximum signal amplitude from transducer 2 was observed on the oscilloscope. This was taken as the zero position and the coordinates were noted. The surface of the foam block was then attached to the compressor using small drops of hot glue at the corners to be able to move the foam along with the compressor. This was done in order to obtain statistically independent pairs of A-lines from transducer 2, while moving the compressor laterally in the x direction. Previous experience has shown that if the foam is not attached to the compressor, the resulting image showed lateral bands resulting from very poor decorrelation of the A-lines as a function of the lateral movement of the compressor.

The compressor along with the foam block were slowly lowered by a predetermined level (precompressed) so that the base of the target made good contact with the table. Transducer 1 was disconnected and transducer 2 was operated in a pulse/echo mode. Transducer 2 was shock-excited once, and the (precompression) echo amplitude signal (A-line) was digitized using a 50 MHz, 8 bit digitizer (Lecroy Corp.). The compres-

![Fig. 3. Panel of images for case 2: length = 60 mm, width = 80 mm. (a) image of stress distribution—analytical model; (b) image of stress distribution—experimental data; and (c) image of the normalized difference between (a) and (b).](image)

![Fig. 4. A schematic diagram of the experimental setup.](image)
sor then was advanced into the elastic medium by a small amount (0.36% of the thickness of the target), and another (postcompression) A-line was acquired. The compressor was then lifted so that the base of the target cleared the table, and laterally advanced by 1 mm so that its axis approached the axis of transducer 2. The process was repeated until the whole plane of interest under the compressor had been interrogated. A total depth of 74 mm was investigated; the lower limit on the depth was set at 8 mm to clear the ring down region of the transducer and also to avoid the highly nonlinear stress regions close to the compressor, and the upper limit on the depth was set at 82 mm. One hundred such A-line pairs were acquired such that the area of interest was 100 mm $\times$ 74 mm for case 1, and 80 A-line pairs were obtained for case 2 to cover an area 80 mm $\times$ 74 mm.

The digitized A-line pairs were then subjected to a piecewise cross-correlation computations to generate strain profiles. The local strain was estimated using the arrival times of the first and second echo sequences from proximal and distal speckle features in the foam, using the following equation (also see Ophir et al. 1991):

$$S(i) = \frac{[t(i) - t(i - 1)]}{2\Delta z/c}, \quad i = 1, n,$$

(2)

where, $S_i$ is the strain estimate for the $i$th segment pair, $t_i$ is the time of arrival of echoes from distal features and $t_{i-1}$ is the time of arrival for echo sequences from proximal features, $\Delta z$ (1 mm) is the longitudinal increment, $c$ (1485 m/s) is the speed of sound and $n$ (74) is the total number of segments in an A-line. The local strain data were displayed in the form of a two-dimensional gray scale image called an elastogram. The bright regions in the image indicate areas of high strain (or stress). Three such elastograms were obtained for each configuration. The foam was moved laterally by about three millimeters relative to the compressor and reattached before obtaining each subsequent elastogram. This was done to obtain statistically uncorrelated elastograms. One composite elastogram was then generated by averaging the three elastograms.

This procedure was used to obtain the strain distribution under a rectangular compressor in two configurations; case 1 when $a = 40$ mm and $b = 30$ mm, i.e., 80 mm long and 60 mm wide, and case 2 when $a = 30$ mm and $b = 40$ mm, i.e., 60 mm long and 80 mm wide.

Figures 2b and 3b show the composite elastograms obtained for each of the two compressors, and Figs. 2a and 3a show the corresponding theoretical longitudinal stress distribution images obtained from eqn (1). Figures 2c and 3c show the two-dimensional gray scale display of the mismatch between the normalized theoretical stress and experimental strain data for cases 1 and 2, respectively, where the bright regions depict the areas of high mismatch between the analytical model and experimental data.

**RESULTS AND DISCUSSION**

The parameter that is actually measured in elastography is the strain, whereas the analytical expressions describe the stress distribution. It is assumed that stress and strain are linearly related within the range of applied strains used in elastography. Although foams show a nonlinear stress/strain relationship over a large range of stress or strain, the behavior could be considered linear for small incremental strains of the order of 1% (Li 1991), as practiced in elastography. Also, the experimental procedure ensures that each scan line pair is obtained at a constant precompression strain. Therefore, the strain data that was obtained experimentally is normalized to the stress data that were obtained analytically, by multiplying it by a constant $k$, which is related to the elastic modulus.

Each of the stress (analytical) and strain (experimental) profiles were low pass filtered to reduce the variance in the estimated strain data (Cespedes and Ophir 1993). The filter was implemented through a 15 point running sum average. The validity of this filtering operation is demonstrated by the fact that it does not have a noticeable effect on the analytical stress profile. This allows us to better evaluate the underlying similarities between the analytical model and the experimental data. Figure 5a shows a typical strain profile (average of three) before and after the filtering operation was performed; Fig. 5b shows the corresponding stress profiles.

The mismatch between the experimental data and the analytical data in the $x - z$ plane is evaluated as follows:

$$\exists = \left\{ \sum_{x, z} \frac{[P(x, z) - kT(x, z)]^2}{[P(x, z)]^2 MN} \right\}^{0.5}.$$

(3)

The value of the constant $k$ for which $\exists$ attains a minimum is computed by setting

$$\frac{\partial \exists}{\partial k} = 0 \Rightarrow k = \left[ \frac{\sum_{x, z} [T(x, z)/P(x, z)]}{\sum_{x, z} [T(x, z)^2/P(x, z)^2]} \right].$$

(4)

where $P(x, z)$ represents the matrix of the magnitude of stress (from eqn (1)) over the entire plane.
of interest, \( T(x, z) \) represents the corresponding strain matrix, \( M \) is the total number of lines investigated and \( N \) is the number of points per line. The \( P(x, z)^2 \) term was introduced in the denominator of eqn (3) so that the computation of \( k \) is not biased by the data close to the compressor where the magnitude of stress, and therefore strain, are higher. The difference between the analytical and the experimental data at any point is now computed as a fraction of the corresponding analytically obtained stress value at that point.

The value of \( k \) thus obtained is then substituted back into eqn (3) to obtain \( \exists_{\text{min}} \), which represents the degree of mismatch between the analytical and the experimental data.

The distribution of mismatch can be displayed in the form of a two-dimensional gray scale image using a matrix form \( \exists(x, z) \), where

\[
\exists(x, z) = \left\{ \frac{[P(x, z) - kT(x, z)]^2}{[P(x, z)]^2} \right\}^{0.5}.
\]  

Two configurations of a rectangular compressor

Fig. 5. (a) a typical experimental stress profile (average of three). (b) corresponding analytical stress profile. Normalized stress is plotted against distance from compressor. Dotted lines indicate profiles before filtering and the solid lines indicates the stress profile after the filtering operation. Positive direction of the x-axis represents decreasing distance to the compressor.
were investigated, as mentioned earlier. Figures 2 and 3 show panels of images for the two cases. It can be observed that there is a good general agreement between the analytical and experimental images. As expected, the images for case 1 show a more uniform distribution of stress over the entire plane as compared to the images for case 2, where there is a marked drop off of stress with increasing depth and also in regions lateral to the axis of the compressor.

The mean error or mismatch $\Xi$ for case 1 was computed to be 22%, while that for case 2 was 18%. The contrast on the error images (Figs. 2c and 3c) was stretched in order to better indicate the distribution of error over the plane of interest. It is apparent that most of the error is not randomly distributed over the plane of interest. There is a bigger mismatch between the analytical model and the experimental data in the regions close to the center of the compressor. We hypothesize that at least a part of the mismatch is due to the inaccurate assumption in the analytical model that the compressor is uniformly loaded, i.e., the pressure applied by each point on the compressor is equal, whereas in the experimental procedure the compressor is uniformly displaced into the foam. It is a well documented fact that uniform displacement of the compressor into an elastic target produces a highly nonuniform distribution of stress over the compressor (Chu and Li 1980; Saada 1974). Uniform displacement of the compressor produces higher stress levels towards the edges of the compressor as compared to the stress levels towards the center of the compressor. The very high stress levels close to the edges of the compressor cause yielding of the target material, thereby causing nonlinear stress/strain behavior. The analytical model to incorporate this nonuniform distribution of stress over the compressor is not, however, trivial. The analytical study of such a situation would be a topic for future work.

**CONCLUSIONS**

We have described an experimental method for studying the planar longitudinal stress field in an elastic medium due to a compressor of arbitrary shape and under arbitrary boundary conditions. Although the technique described here is presented as a tool to study the correction needed for an artifact in elastography, the generality of such a technique is evident. In principle, such a technique could be used to study the distribution of longitudinal stress under a loaded compressor of arbitrary shape and size. Lateral stresses could also be studied by using a transducer mounted near the side of the target without actual contact with it. A volumetric study of the stress distribution can also be performed by scanning different planes. This technique could be speeded up considerably by using a linear array transducer instead of transducer 2, and digitizing the entire data in one operation.

The boundary conditions, such as the size and shape of the compressor and the surface on which the target is placed, affect the stress distribution in the target. The experimental stress mapping technique is useful in studying the influence of the various boundary conditions on the stress distribution in the target to allow first order corrections of elastograms to obtain more quantitatively significant images. An earlier paper (Ponnekanti et al. 1992) described the stress distribution in an elastic target, along the axis due to two opposing, coaxial, circular compressors. It was observed that there was a very good agreement between the measured stress profile and the corresponding analytically predicted stress profile, which took into account the boundary conditions. This paper extends the experimental technique to imaging the longitudinal stress distribution in a two-dimensional scan plane. It is to be noted that the analytical model used here accounts for the two-dimensional geometry of the compressor as well.

**Acknowledgement**—Supported in part by National Institutes of Health Grant #ROI-CA38515 and by Diasonics Inc, Milpitas, CA.

**REFERENCES**


Li, X. Measurement of Young’s modulus. Technical memorandum #97. Ultrasound Laboratory, Department of Radiology, University of Texas Medical School at Houston, 09/90; (unpublished).


