Original Contribution

FUNDAMENTAL MECHANICAL LIMITATIONS ON THE VISUALIZATION OF ELASTICITY CONTRAST IN ELASTOGRAPHY

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Abstract—Elastography is a new ultrasonic imaging technique that produces images (elastograms) of the elastic properties of compliant tissue. To determine the Young’s modulus it is necessary to measure or estimate any five of seven relevant variables. In elastography, the measured quantity is the normal strain component in the direction of the applied load, and the three normal components of stress may be estimated using the modified Love’s analytical models while assuming a value close to 0.5 (incompressible) for Poisson’s ratio. The distribution of Young’s moduli can thus be computed and displayed in the form of two-dimensional images called elastograms. The analytical models used for the estimation of the three normal components of stress assume that the target is semi-infinite and homogeneous in composition. The objective of this article is to determine some of the errors associated with the assumption of homogeneity of the target. Experiments using computer simulations were performed to study the efficiency with which elastograms display the contrast in the Young’s modulus of a lesion or target, with respect to its background under certain conditions. It was observed (using the definition of contrast-transfer efficiency of elastography as the ratio of the elasticity contrast as measured from an elastogram, to the true contrast) that elastograms were consistently efficient in quantitatively depicting the elasticity contrast of hard lesions; however, they showed suboptimal contrast-transfer efficiency in cases of soft lesions in a hard background. In general, elastograms are efficient in displaying the elasticity contrast of hard or soft lesions which have a low contrast level with respect to the surroundings, irrespective of their size and location.

Key Words: Ultrasound, Elastography, Elastogram, Stress, Strain, Poisson’s ratio, Contrast, Young’s modulus, Finite element analysis, Scattering, Contrast-transfer efficiency.

INTRODUCTION

Changes in tissue elasticity are generally correlated with pathological phenomena. Many cancers, such as scirrhouus carcinoma of the breast, appear as extremely hard nodules, which are a result of increased stromal density. However, the small size of the pathological lesion and/or its deep location in the body hinders its detection and evaluation by ultrasound, especially when its acoustic backscatter properties are similar to those of the embedding normal tissue.

In a previous article, we described a technique known as elastography for imaging the elasticity of biological soft tissues. Elastography (Ophir et al. 1991) is a new ultrasonic imaging technique, which produces images of the internal strains produced in a soft-tissue target resulting from a small external displacement of the tissue. Elastography is performed by obtaining a set of ultrasonic RF A-lines from a target, subjecting the target to a small deformation and obtaining a second set of RF A-lines. Time-shift estimations along the direction of the applied loads are computed by performing piecewise cross-correlation on congruent pairs of A-line segments. The time-shift estimations are then converted to strain information which could be displayed in the form of a two-dimensional strain image.

Strain by itself is an incomplete indicator of the actual elasticity distribution in the target. Although the spatial variation in strain may partially reflect the changes in local Young’s moduli, the knowledge of the distribution of stresses is essential to characterize the true contrast in elasticity (Ophir et al. 1991). Ponnekanti et al. (1994) have shown a technique for the estimation of the stress distribution in an isotropic, homogeneous, finite elastic target that is being deformed by a rectangular punch using a modification of Love’s (1929) analytical models. The estimated distri-
bution of stresses is used along with the measured values of strain, while assuming the target tissue to be nearly incompressible (Poisson's ratio $\approx 0.5$), to estimate the distribution of Young's moduli in the target. The distribution of Young's moduli in the plane of interest is then displayed in the form of a two-dimensional gray-scale image known as an elastogram.

The modified Love's models that are used to determine the stress distribution assume that the target is homogeneous and semi-infinite. Since real targets are seldom homogeneous or semi-infinite, the elastograms suffer from certain errors. Errors incurred due to the assumption of target homogeneity are investigated in this study.

It has been observed from preliminary simulation data that the lesion-to-background elasticity contrast of elastographic lesions changes with the location and with the true elasticity contrast. The principal objective of this article is to characterize these changes under a restricted set of experimental conditions of target composition and boundary conditions. The problems associated with building accurate acoustomechanical phantoms preclude their use for these experiments, since it is required that the mechanical as well as the acoustic properties be independently controllable in the same phantom. Therefore, the development of a software tool to simulate elastograms while allowing control of various mechanical and acoustical parameters was important to this study. Simple one-dimensional models have been used previously by Céspedes and Ophir (1993) and Belaïd et al. (1994) to simulate elastograms. The use of these models was restricted to simple applications where detailed analysis of the mechanical response of the target to complex boundary conditions was not essential. Therefore, these models were not adequate to accurately describe the complex three-dimensional nature of the mechanical problem in elastography. For example, experimental results in the past have revealed several artifacts associated with the three-dimensional nature of the mechanical response of a target to external boundary conditions, such as the nonuniform distribution of stress in an elastic target that is being acted upon by a finite size punch (Ponnnekanti et al. 1992). Furthermore, the experiments of Belaïd et al. (1994) involved very low contrast lesions. Such lesions are expected to minimally interact with the background, resulting in excellent contrast-transfer efficiency (defined as $\eta = \text{observed elasticity contrast}/\text{true elasticity contrast}$). In the present work, we investigate this efficiency of elastography over a much wider, $\pm 40$ dB, contrast dynamic range. The results reveal a markedly different behavior of the contrast-transfer efficiency of hard and soft lesions, while confirming the excellent contrast-transfer efficiency asso-

ciated with low contrast lesions. These are expected results that stem from some fundamental mechanical limitations affecting elasticity imaging, which are discussed later.

In recent years, finite element analysis (FEA) techniques have found widespread use in the study of the mechanical response of biological tissues (Beaupre and Carter 1991). FEA methodologies have been incorporated directly into prosthetic design for limbs by Krousop et al. (1987), Steege et al. (1992) and Sanders et al. (1993). Vannah (1990) has studied the mechanical response of bulk muscular tissue in vivo using FEA techniques and corroborated the results using indentor tests. Parker et al. (1990), Céspedes (1993) and Ponnnekanti et al. (1994) have previously used FEA to study the mechanical response of soft tissue models to gain insight into the mechanical considerations that are involved when measuring tissue elasticity using ultrasound.

An FEA computer program allows the generation of realistic mechanical models that take into account the complex three-dimensional motion encountered in elastography. The addition of a custom program capable of superimposing an acoustic model on an FEA model has resulted in a powerful tool. It offers a convenient method to simulate elastograms that demonstrate realistic mechanical behavior while allowing control over the mechanical properties, composition of the target and the boundary conditions. Additionally, various acoustic and signal processing parameters could also be easily implemented. Elastograms created using this simulation were used to perform experiments to evaluate the contrast-transfer efficiency of elastography under a restricted set of conditions. Elastograms were simulated with circular lesions possessing various diameters, lesion-to-background elasticity contrast and locations with respect to the transducer. The contrast-transfer efficiency in the estimation of the lesion-to-background elasticity was computed by comparing lesion-to-background contrast in the elastograms against the contrast that was used as the input to the FEA program.

Although we describe a tool that is capable of simulating realistic elastograms, the aim of this study is the analysis of the mechanical aspects of elastography only. Therefore, analysis of contrast-transfer efficiency was performed on elastograms derived from the FEA models, which did not contain noise due to precision errors in time-delay estimations described by Céspedes and Ophir (1993). This is a valid approach since Belaïd (1993) has shown these errors to have a zero mean. The results of such analysis must therefore be regarded as the best that are theoretically achievable; realistically, the presence of small lesions (on
the order of 1 to 2 mm) may be totally obscured by
noise using current algorithms. However, various noise
reduction algorithms currently under development are
expected to result in elastograms that are of a quality
which is much closer to the theoretically achievable
level (Céspedes and Ophir’ 1993).

**THEORY**

Linear elastic materials obey the generalized
Hooke’s law which states that stress is proportional
to strain. The general form of the law is expressed by the
statement: each of the components of the state of stress
\( \sigma \) at a point is a linear function of the components
of the state of strain \( \varepsilon \) at that point. Mathematically,
this is expressed as (Saada 1989):

\[
\sigma_{kl} = C_{klmn} \varepsilon_{mn} \tag{1}
\]

where the \( C_{klmn} \) are elastic constants comprising the
elements of a stiffness matrix \( C \). In general, there are
81 such constants corresponding to the indices \( k, l, m, n \)
taking values equal to 1, 2 and 3. The symbol
\( \sigma_{kl} \) is read as the stress component in a plane perpendi-
cular to the axis indicated by the index \( k \) and along the
axis indicated by the index \( l \). A similar notation has
been used to describe the strain, where \( m \) and \( n \) have
been used to show that a stress component in any
direction can be related to the strain component in any
other direction through an appropriate elastic constant,
which is the reason for the four subscripts for \( C_{klmn} \).

By assuming that the material is homogeneous and
isotropic, the number of independent constants re-
quired to completely characterize the elastic properties
of the material reduces to two (Saada 1989). These
constants are the Lame’s constants, \( \lambda \) and \( \mu \) (shear
modulus). The stress–strain relationship for an iso-
 tropic, homogeneous material can now be written in
the index notation as (Saada 1989):

\[
\sigma_{kl} = 2\mu \varepsilon_{kl} + \lambda \delta_{kl} \varepsilon_{mn} \tag{2}
\]

where \( \delta_{kl} \) is Kronecker’s delta, which is unity when \( k = l \) and zero otherwise. Rearranging and rewriting
the normal (non-shear) terms of eqn (2) in the extended
form in terms of stress, strain and the engineering pa-
rameters Poisson’s ratio (\( \nu \)) and Young’s modulus
\( E \), we have (Saada 1989):

\[
e_{11} = \frac{1}{E} [\sigma_{11} - \nu \sigma_{22} - \nu \sigma_{33} ] \tag{3a}
\]

\[
e_{22} = \frac{1}{E} [\sigma_{22} - \nu \sigma_{11} - \nu \sigma_{33} ] \tag{3b}
\]

\[
e_{33} = \frac{1}{E} [\sigma_{33} - \nu \sigma_{11} - \nu \sigma_{22} ] \tag{3c}
\]

where the duplicated subscripts indicate normal com-
ponents in the corresponding directions, and where:

\[
\nu = \frac{\lambda}{2(\lambda + \mu)} \tag{4a}
\]

and

\[
E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}. \tag{4b}
\]

From eqn (3a–c) it can be seen that, to compute
the Young’s modulus of any element inside a target,
at least five out of the seven variables must be known.
Since most tissues can be treated as incompressible
(Krouskop et al. 1987), a value close to 0.5 can be
assumed for the Poisson’s ratio (\( \nu \)), thus reducing
the number of variables which must be known to four.

It should be noted that the stress components cannot
be measured directly, and must therefore be esti-
mated. The problem of estimating the various compo-
nents of stress and strain in a semi-infinite, homo-
geous, elastic target that is subjected to a deformation
using an externally applied load is known as the Bous-
series problem (named after Boussinesq 1885). A spe-
cial case of the problem is the deformation of the target
using a rectangular punch. This problem has been in-
vestigated in detail by Love (1929), who has devel-
oped mathematical expressions to describe the spatial
distribution of the various components of stress as a
function of the punch dimensions and the material
properties for homogeneous, isotropic, semi-infinite
materials. His expressions for the normal components
of stress may be written in their functional form as:

\[
\sigma_{11} = \sigma_{11}(\sigma_{L}, x_{1}, x_{2}, x_{3}, a, b) \tag{5a}
\]

\[
\sigma_{22} = \sigma_{22}(\sigma_{L}, x_{1}, x_{2}, x_{3}, a, b, \nu) \tag{5b}
\]

\[
\sigma_{33} = \sigma_{33}(\sigma_{L}, x_{1}, x_{2}, x_{3}, a, b, \nu) \tag{5c}
\]

where \( \sigma_{L} \) is load applied by the punch and which can-
cels out in contrast calculations, \( x_{1}, x_{2} \) and \( x_{3} \) are the
coordinates of the location of interest (Fig. 1) and \( a \)
and \( b \) are the length and width of the rectangular punch,
respectively. The expressions in their explicit form are
cumbersome and have been described completely by
Love (1929).

In the more practical case, where the dimensions
of a linearly elastic target are finite along the \( x_{1} \) axis,
the principle of superposition could be used to modify
superposition involving the top and the bottom boundaries can be derived from eqn (6) (Ponnekanti et al. 1992). Elastography in homogeneous targets could then be described by eqn (3a), where $\varepsilon_{11}$ is measured ultrasonically, the values for the three components of normal stress are estimated using eqn set (6), and a value close to 0.5 is assumed for the Poisson’s ratio (Krouskop et al. 1987).

The simplifying assumptions built into the modified Love’s theory gives rise to two types of errors. The first is due to the finite dimensions of the elastic target along the transverse $x_3$ direction which are assumed infinite, and the second is due to the presence of elastic inhomogeneities in the target which is assumed homogeneous. Together, these appear to be responsible for a reduction of the observed elastic contrast in elastograms as compared to the true contrast. It is to be noted that Love’s model makes the assumption that the punch is uniformly loaded, while elastography is performed by uniformly displacing the target surface through a small distance which results in a nonuniform loading of the punch (Saada 1989). However, this discrepancy in the boundary conditions causes errors only in the regions which are very close to the punch surface, and are negligibly small in deeper regions in the target (Love 1929).

For elastograms to represent the distribution of the absolute Young’s moduli, it is necessary to measure the load applied by the punch ($\sigma_L$). For a uniformly loaded punch, as is the case in Love’s model, it is simply a multiplicative factor in eqns (6) that scales the distribution of stress inside the target. However, since this article investigates the efficiency of elastography in detecting the contrast in elasticity of a lesion with respect to its background, the value of $\sigma_L$ is unimportant, and was ignored.

**MATERIALS AND METHODS**

Elastograms were generated using several steps. First, mechanical models were generated using a commercial finite element analysis program (MSC-PAL2, Version 4, McNeal-Schwendler Corp., Los Angeles, CA). An acoustic Rayleigh-scattering model was added to the displacement information obtained as an output of the FEA program (see Appendix). Pre- and postloading RF-signals were generated using a convolutional model and a simulated acoustic transducer (Céspedes and Ophir 1990). The strain distribution was estimated by applying cross-correlation techniques to congruent pairs of these RF-signals (Ophir et al. 1991). Modified Love’s models were applied with appropriate boundary conditions to estimate the distribution of all three normal components of stress at all
locations. The strain and stress data were then converted to Young’s moduli and displayed as elastograms.

The FEA mechanical models were generated with some typical conditions for clinical elastography in mind. The punch used for the application of the load was chosen to have a lateral dimension of one fifth the lateral dimension of the target (0.2a), where a is the lateral dimension of the target, and the depth of the target was chosen to be equal to 3a. These dimensions are typically encountered in elastography of the breast. The punch at the base of the target was assumed to have an area larger than the area of the base of the target as shown in Fig. 1. These boundary conditions were used to simulate a situation where the breast is placed on a large flat table while being examined by a small rectangular linear array transducer (Céspedes et al. 1993).

The entire target was simulated with a total of 1353 nodes (Fig. 1). In accordance with common practice (Beaupre and Carter 1991), the number of nodes per unit area was varied over the target model as shown in Fig. 1, for optimum utilization of the number of nodes available in the FEA program. A higher nodal density was used in areas where the distribution of stresses was expected to be more nonuniform, such as in areas close to the punch and in the area of the lesion. A mesh with quadrilateral elements was generated to connect the nodes. FEA techniques require that the choice of the nodal locations and meshing be made carefully to have a stable model. Therefore, simple trial models were initially generated with different nodal and mesh arrangements to check for the stability of the model. The experimental target was then simulated with a similar arrangement of nodes and elements.

The mechanical problem in elastography is scalable in terms of the punch size, just as in sonography, the acoustic problem may be scaled in terms of the wavelength. Therefore, the distances were calibrated in terms of an arbitrary constant called a “unit” such that the punch had a lateral dimension of 80 units. The target contained a rectangular arrangement of quadrilateral elements that was 400 units wide and 240 units deep (Fig. 1). It is to be noted that a unit is not necessarily equal to the distance between nodes; it was a constant that was arbitrarily chosen to scale the distances in the models. The background of the target had a Young’s modulus of 21 kPa that was arbitrarily chosen to represent normal tissue, and a Poisson’s ratio of 0.495. Since it has been shown (Céspedes and Ophir 1993) that strains larger than 1% to 2% may cause decorrelation artifacts, a small vertical deformation of 1 unit (0.4% compressive strain) was provided by a punch. The nodes corresponding to the lower surface were restricted to move in the lateral direction only while the sides were left free (Fig. 1). Circular lesions were simulated inside the target by nodes and elements arranged in concentric circles. Two sizes for the circular lesion were chosen; the diameters of the smallest and the largest lesions were 1/20 (4 units) and 1/2 (40 units) the lateral dimension of the punch, respectively (see Appendix). Given a typical compressing aperture size of 40 mm reported by Céspedes et al. (1993), these values correspond to lesion diameters of 2 mm and 20 mm, respectively. Six locations were chosen for the center of the lesion at three different depths, three were along the axis of the punch and the other three were along an axis parallel to and 20 units to the right of the punch axis. The locations for these lesions were chosen in a region under the punch which corresponds to the insonified region of interest (Fig. 2). The depths of the lesions were chosen as 40 units, 120 units and 200 units to represent situations where the lesions are within distances that are equal to 0.5a, 1.5a and 2.5a. Symmetry about the axis of the punch was assumed. Six different lesion-to-background elasticity contrasts were chosen: 0.01:1 (−40 dB), 0.1:1 (−20 dB), 0.5:1 (−6 dB), 2:1 (6 dB), 10:1 (20 dB) and 100:1 (40 dB). The theoretical contrast of 1:1 (0 dB) was also considered.

The nodal coordinates before and after the application of the load for each case were obtained as an output from the FEA program. To facilitate the display of the data in the form of two-dimensional images, the nodal location data from the irregular nodal pattern were first mapped onto a regular rectangular nodal pattern (Appendix). Two sets of data with regular nodal patterns were thus generated, one each for the pre- and postloading target. A convolutional acoustic model as described by Céspedes and Ophir (1990) was then superimposed on the data sets for the pre- and postloading target by using a model for a sampled aperture transducer with a known impulse response, and a model for a Rayleigh scattering medium. A large collection of randomly placed point scatterers was simulated in the region of interest and superimposed on the unloaded nodal data set. Simulated RF echo sequences were constructed by adding up the impulse responses of the transducer elements at the location of every scatterer. The same collection of scatterers was then superimposed on the loaded nodal data set. The displacement of each scatterer was computed by linear interpolation of the displacements of its four nearest neighboring nodes using the same linear interpolation model that was used to map the irregular nodal pattern to a regular pattern. This simulated a realistic situation of two-dimensional scatterer motion in a target that
has been subjected to a deformation by a punch of finite size while the motion of scatterers in the third dimension was constrained. A second set of RF echo sequences was generated from these displaced scatterers. The strain was then estimated from the gradient along the $x_1$ axis of the time shift measurements using crosscorrelation techniques on congruent segments of pre- and postloading RF echo sequences (Ophir et al. 1991). The target was assumed to be lossless and to have a constant speed of sound. Figure 3 illustrates the results at three stages in the simulation of a typical elastogram. Figure 3a is a strain image of a homogeneous target as computed from the output of the nodal displacement from the FEA program (see Appendix). Fig. 3b shows an image of strain as computed using the time-delay estimations from RF echo sequences, and Fig. 3c shows the corresponding elastogram.

The three normal stress components needed for elastogram generation were computed using eqn (6) where $A_1$ was taken as the area of the punch ($80 \times 2000$ units) and $A_2$ was taken as the area of the base of the target ($400 \times 2000$ units) for a target with Young's modulus of 21 kPa and Poisson's ratio of 0.495. The out-of-plane dimension was chosen to be very large (2000 units) to reduce the normal stress and strain components in that direction (along the $x_3$ axis) to negligibly small values (two orders of magnitude lower). The three normal components of stress were substituted into eqn (3a) along with the measured values of the normal component of strain, to obtain the Young's moduli distribution over the region of interest. The Young's moduli data were then displayed as elastograms. Figure 3c shows a typical elastogram.

Although to this point we have described a method to generate realistic elastograms, the aim of this article is to study the purely mechanical aspects of elastography. Realistic elastograms contain noise related to several acoustic and signal processing parameters as described by Céspedes and Ophir (1993). During earlier studies it was observed that the presence of this noise in the elastograms limited the detection of small lesions (Belaid et al. 1994). For larger lesions, the mean of the noise in the elastograms was close to zero. Therefore, valid mechanical analyses could be conducted on images that did not contain any noise related to the acoustic and signal processing parameters. This was done by simply computing the exact strain distribution along the $x_1$ axis as the gradient of the interpolated nodal displacements from the FEA program, instead of measuring the strain distribution using the acoustic model. The strain distribution was then used to determine the distribution of Young's moduli as described earlier. Figure 4 shows a typical example of the problem of detectability. Two pairs of elastograms are shown, one each for a small and a large lesion. Each pair consists of one elastogram derived from the FEA model, and another one that adds the acoustic model to it. Note that the small lesion (Fig. 4a and b) is detectable only in the ideal, noiseless elastogram. However, the large lesion (Fig. 4c and d) is detectable in both elastograms.

An image processing program (ImagePro Plus, Media Cybernetics, Silver Spring, MD) was used to display and analyze the images. A limited range of gray levels (256) was available in the program for the display of the elastograms. Since the range of Young's
modulus contrast that was used for this study was large, a linear allocation of gray-levels was not feasible. Therefore, the images were displayed in terms of the reciprocal of the Young's moduli (in units of kPa$^{-1}$). Figure 3c shows such an image.

All the images were displayed using gray-level
allocation between the same maximum and minimum values to allow for a valid comparison among images. Once the images were displayed, the mean gray-level of the background was measured. The mean gray-levels inside the larger lesions were measured from the image by using the data from inscribed rectangular windows. The gray-level measurement for the small lesions was determined by noting the minimum value in a gray-level profile along a horizontal line that was drawn through the center of the lesion. The size of the small lesions precluded the use of spatial averaging of gray-levels. The contrast in the Young’s moduli for each case was then determined as the reciprocal of the ratio of the gray-level corresponding to the background to that of the lesion. The precision in the measurements was about ±5% of the mean. The results are plotted in Fig. 5a–f.

RESULTS

We define the contrast-transfer efficiency of elastography as the ratio of the contrast in elasticity as measured from an elastogram, to the true contrast in the corresponding target, viz.:

\[ \eta = \frac{C_0}{C_i} \]  

where \( C_0 \) is the observed contrast, \( C_i > 0 \) is the true contrast and \( \eta \) is the contrast-transfer efficiency (\( \eta = 1 \) indicates an ideal 100% efficiency; \( \eta = 0 \) indicates 0% efficiency).

Since we have investigated a wide dynamic range, i.e., 0.01 \( \leq C_i \leq 100 \), it is convenient to represent the results on a logarithmic decibel (dB) scale. By taking the logarithm of both sides of eqn (7), we get:

\[ \eta(\text{dB}) = C_0(\text{dB}) - C_i(\text{dB}). \]  

Using this presentation, \( \eta = 0 \) (dB) means 100% efficiency, and \( \eta < 0 \) (dB) signifies a reduced efficiency.

The results are plotted in Fig. 5a–f, which shows plots of the observed lesion elasticity contrast from the elastograms versus the true contrast on a dB scale. A separate plot was drawn for each of the six locations of the lesion. Each plot contains two curves, one each for the small (4 units in diameter) and large (40 units in diameter) lesions. A diagonal line was drawn to represent ideal performance (\( \eta = 0 \) [dB], or \( C_0 = C_i \)).

Figure 5a–f clearly demonstrates that the behavior of the observed contrast is different for hard and soft lesions in all cases considered. To get a clearer picture, we combine all the data shown in each part of Fig. 5 and plot in Fig. 6 the mean contrast-transfer efficiency \( \overline{\eta} \) (in dB) versus true contrast \( C_i \), where the error bars represent 1 standard deviation. This figure clearly shows several interesting results:

1. For hard or soft lesions with low contrast (\( |C_i| \leq 6 \) dB), the mean contrast-transfer efficiency \( \overline{\eta} \) of elastography is always better than \( -3 \) dB (\( >0.7 \)).
2. For soft lesions with high contrast (\( C_i < -6 \) dB), the mean contrast-transfer efficiency gets progressively poorer as the contrast increases, achieving a very low mean transfer efficiency of \( -33.4 \) dB (\( \sim 0.02 \)) at a contrast level \( C_i = -40 \) dB.
3. For hard lesions with high contrast (\( C_i > 6 \) dB), the mean transfer efficiency declines to a moderate fixed level of about \( -5 \) dB (\( \sim 0.56 \)) for lesions with a contrast level of 20 to 40 dB.
4. These results were consistent for all cases of lesion size and location, as demonstrated by the modest values of the standard deviations (shown in Fig. 6), which were on the order of \( \pm 1 \) to 2 dB.

DISCUSSION AND CONCLUSIONS

The major factor that influences the continuously declining contrast-transfer efficiency of elastography of soft lesions is the phenomenon by which soft lesions that are confined by a harder background appear harder than they really are, when the Poisson’s ratio is non-zero (Hueter and Bolt 1955). This results in an effective contrast-dependent hardening of the constrained lesion. This phenomenon represents a fundamental limitation on the elastographic imaging of soft lesions. For soft lesions with low contrast (\( C_i > -6 \) dB), the mean contrast-transfer efficiency is quite good (\( \sim -3 \) dB).

On the other hand, for all cases that were investigated, the visualization of hard lesions had efficiencies which were consistently better than about \( -5 \) dB. It is also interesting to note that for contrast levels \( \geq 20 \) dB, the mean transfer efficiency remains at this \( -5 \)-dB contrast level, which raises the possibility of applying a constant correction factor to the observed contrast level, in order to correct it for this bias. For low contrast levels (<6 dB), the contrast-transfer efficiency is rather good, ranging between 0 and \( -3 \) dB. We postulate that the source of this behavior is the perturbation of the background by the lesion, which is not accounted for by Love’s modified model that assumes a uniform target. Indeed, the data of Fig. 6 demonstrate that for hard lesions of low contrast, the contrast-transfer efficiency is high (i.e., \( C_0 \approx C_i \)); these cases would be expected to cause the least amount of perturbation of the background, and hence, be more efficient. For hard lesions with high contrast (\( C_i \geq 20 \) dB), the lesions would tend to behave as rigid bodies, exerting
Fig. 5. Plots of transfer characteristics (observed contrast versus true contrast) for various locations of large and small lesions. The diagonal line indicates ideal performance. (a) Location 1—along the axis, depth = 40 units (0.5a). (b) Location 2—along the axis, depth = 120 units (1.5a). (c) Location 3—along the axis, depth = 200 units (2.5a). (d) Location 4—20 units right of the axis, depth = 40 units (0.5a). (e) Location 5—20 units right of the axis, depth = 120 units (1.5a). (f) Location 6—20 units right of the axis, depth = 200 units (2.5a).
Fig. 6. Mean contrast-transfer efficiency $\bar{\eta}$ as a function of the lesion-to-background contrast ($C_t$) for visualization of all soft and hard lesions of both sizes in all locations. Error bars represent ±1 SD.

a fixed perturbation on the softer background material regardless of contrast; this would tend to result in a fixed bias in the efficiency such as that which is demonstrated in Fig. 6. It is interesting to note that the contrast-transfer efficiency of elastography is generally high for hard lesions, in light of the fact that most cancerous lesions tend to be harder than normal tissue.

In conclusion, we have described elastography in the context of the basic theory of elasticity and have listed the steps involved in the determination of the local Young’s moduli in an isotropic, elastic target. Well-known analytical models have been adapted with appropriate modifications to estimate the distribution of stress. We have presented a powerful software simulation tool that is capable of simulating realistic elastograms. In addition to being easy to use, it allows independent control of the various acoustoelastic parameters that are involved in elastography. The aim of this study was to analyze the mechanical aspects of elastography and, for this reason, the experiments were performed on elastograms that did not contain the zero mean noise that results from the precision errors involved in the time-delay estimations in strain measurement (Belaïd et al. 1994; Céspedes and Ophir 1993). In realistic situations, in which the elastograms contain noise due to such precision errors, small lesions may not be detectable. For these cases the results presented in this article are not strictly applicable, and should be regarded as ideal. We have performed several practical studies using simulated elastograms and have evaluated the results in terms of a newly defined contrast-transfer efficiency parameter. It was noted that there exists a fundamental limitation on the ability to accurately image the contrast of soft lesions due to the constraining effects of the background. For hard lesions, a mean contrast-transfer efficiency of $\bar{\eta} > -5$ dB appears reasonable in view of the large dynamic range of tissue elasticity that spans at least a 40-dB range in soft tissues such as the breast (Sarvazyan 1993). The combined effect of lesion size and location on the efficiency was small (on the order of ±1 to 2 dB).

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APPENDIX

Circular lesions were chosen for this investigation since it is known from clinical data that most lesions have rounded shapes. There could be two ways to generate circular shapes in the FEA program, either by generating the nodes in a cylindrical coordinate system or to generate a fine rectangular mesh in the region of the lesion and approximate a circle. Preliminary test models showed that the latter model resulted in artificial stress concentration regions at the corners of the rectangles, thus giving erroneous results. Therefore, localized cylindrical coordinates were used to generate the circular lesions. Ten concentric nodal circles were generated, so that lesions with five different diameters could be selected by changing the properties of two circles at a time. All the nodes were meshed using quadrilateral elements since they give a stable FEA model. The model could get unstable for quadrilateral elements inside the smallest circle due to the convergence of those elements toward triangular shapes as the center of the circle is approached. This problem is overcome by incorporating several (10) progressively smaller circles inside the circle corresponding to the smallest diameter for the lesion, so that the rate of convergence of the quadrilateral elements is reduced (Fig. 1), thus increasing the stability of the model.

As a result of the restrictions on the nodal and meshing arrangements that are described above, the output of the FEA program contains displacement data at irregularly arranged nodes. To display the data as images, it is critical that the data be arranged in a rectangular matrix form. Therefore, irregularly arranged sets of pre- and postloading nodal displacement data that are produced by the FEA program are mapped into a regular rectangular grid using linear interpolation.

The interpolation was done using an algorithm that first reads the coordinates and the deformation of the nodes from the output of the FEA simulation. The size of the rectangular interpolation matrix was chosen as 241 rows by 401 columns, corresponding to the depth and the lateral dimension, respectively. The interpolation matrix was mapped onto the output of the FEA program such that the coordinates of the points corresponding to the corners of the target were coincident. The region around each point on the interpolation matrix was divided into four quadrants and a nearest neighboring node location was determined as shown in Fig. A1. The quadrilateral in solid lines indicates an element connecting the nodes (solid circles) a, b, c and d as obtained from the output of the FEA program. The thin lines represent the grid lines connecting the points (hollow circles) of the interpolation matrix. The displacement of the point o1 is determined by linear interpolation of the displacement of points a and b (Fig. A2) using the expression

\[ D_{o1} = \frac{D_a^{o1} + D_b^{o1}}{l1 + l2} \]  

(A1)

where D is the change in the coordinates of the point indicated by its subscript, and l1 and l2 are the distances of points a and b from o1, respectively, as shown in Fig. A2. The displacement of the point o2 was calculated by linear interpolation between displacements of points c and d with appropriate modifications of eqn (A1). Finally, the displacement of point o was obtained from the displacements of points o1 and o2. A rectangular matrix of 401 × 241 points was thus generated to represent the entire target. The region of interest was, however, taken as 81 × 241 points representing the “insonified” region (boxed area in Fig. 2). A similar algorithm for interpolation was used to determine the displacement of each scatterer after a cloud of randomly placed scatterers was superimposed on the rectangular grid. The location of each scatterer was compared against the points on the rectangular grid and the four nearest points were determined. The displacement of the scatterer was then computed as a linearly weighted mean of the displacements of the four nearest neighboring points.

Fig. A1. Irregular mesh arrangement in FEA models (bold lines) and a regular arrangement into which the motion is mapped (thin lines).

Fig. A2. Representation of linear interpolation between two points (see text).