REDUCTION OF SIGNAL DECORRELATION FROM MECHANICAL COMPRESSION OF TISSUES BY TEMPORAL STRETCHING: APPLICATIONS TO ELASTOGRAPHY

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Abstract—Elastography is based on the estimation of strain due to tissue compression. Strain is computed from the estimates of time delays between gated precompression and postcompression echo signals. Time delay estimates are obtained from the location of the peak of the crosscorrelation function between gated precompression and postcompression signals. It is of paramount importance to accurately estimate the time delays for good quality elastograms. A main source of time delay estimation (TDE) error in elasticity imaging is the decorrelation of the echo signal as a result of tissue compression (decorrelation noise). The effect of decorrelation on the mean of the cross-correlation function and the correlation coefficient has been investigated. The expected value of the cross-correlation function between the precompression and postcompression signals was shown to be a filtered version of the autocorrelation function of the precompression signal. In this article, the effect of temporal stretching of the postcompression echo signal on the cross-correlation function will be investigated along the same line. The applied compression is assumed to be uniform; the decorrelations introduced by the lateral and elevational tissue movements are ignored. The theory predicts that if the postcompression echo signals are stretched before the TDE step, then for small strains, the cross-correlation function very closely resembles the autocorrelation function. For larger strains, correlation is improved if temporal stretching is applied. The theory is corroborated by results from simulation and homogeneous phantom experiments. Thus, the decorrelation noise in elastograms can be reduced by temporal stretching of the postcompression signal.


INTRODUCTION

Over the past several years, ultrasonic imaging methods based on tissue elasticity have gained significance for diagnosis of disease (Alam et al. 1994; Krouskop et al. 1987; Lerner and Parker 1987; Lerner et al. 1990; O'Donnell et al. 1994; Ophir et al. 1991; Yamakoshi et al. 1987; Yamakoshi et al. 1990). Magnetic resonance imaging has also been used (Fowlkes et al. 1995; Mithupillai et al. 1995; Plewes et al. 1995). Ultrasonic techniques for the estimation of tissue elasticity are based on the estimation of strain due to the compression of the tissues (O'Donnell et al. 1994; Ophir et al. 1991). The local tissue displacements are estimated from the time delays of gated precompression and postcompression echo signals, which are then used to estimate the axial strain. In elastography (Ophir et al. 1991), time delays are estimated from the location of the peak of the crosscorrelation function between the precompression and postcompression echo signals. The strain estimation techniques track the motion of speckle scatterers; however, as the scatterers become resolvable (Varghese and Donohue 1994; Varghese and Donohue 1995), the accuracy of the displacement estimate improves.

The quality of elastograms is highly dependent on the quality of the strain estimation, which in turn is dependent on the quality of time delay estimation (TDE). TDE in elastography is mainly corrupted by
two factors. First, the random noise (electronic and quantization) introduces some errors in the TDE. Second, even when the signal-to-noise ratio (SNR) in the echo signals is high, the decorrelation of the echo signal as a result of tissue compression (decoration noise) can significantly corrupt the TDE. When tissue is compressed, unresolved scatterers generally move closer together along the axial direction, resulting in a signal that can be quite different from that obtained in the precompression state. Additionally, scatterers move in the lateral and elevational directions, resulting in further signal decorrelation.

Céspedes (1993) discussed the effect of tissue compression on decorrelation. A theoretical model was presented that described the mean changes of the cross-correlation function and the correlation coefficient with axial strain. The decorrelation model for axial strain described the expected value of the cross-correlation function as a filtered version of the autocorrelation function of the precompression signal.

Wear and Popp (1987) described a method to characterize cardiac contractile performance from measurements of temporal correlation properties of ultrasonic tissues using the variance of the particle velocity as an indicator of contractile performance. Meunier and Bertrand (1995) have investigated the effect of tissue rotation, translation and biaxial deformation on ultrasound images. Expressions for the decorrelation of radio frequency and demodulated echo signals were derived.

In this article, we investigate the effect of temporal stretching (Céspedes and Ophir 1993) of the post-compression echo signals prior to the cross-correlation step, on the behavior of the cross-correlation function. The theory is validated using one-dimensional simulation and homogeneous phantom experiment. The validity of the analysis presented here is restricted to one-dimensional uniform tissue strains (we ignore the decorrelation introduced by the lateral and elevational components of strain). The assumption that the lateral and elevational decorrelations are small compared to the axial decorrelation is reasonable when the target is compressed with a compressor that is large compared to the transducer (Ponnekanti et al. 1992), and the resulting strain results in lateral and elevational displacements that are small compared to the spatial resolution of the ultrasound system in the corresponding direction. The effects of lateral and elevational decorrelation on the estimator performance will be addressed in future studies.

THEORY

Crosscorrelation function

Various methods can be used to obtain an estimate of the time delay between a broad-band signal and a delayed replica of the signal, in the presence of additive noise. Locating the maximum of the cross-correlation function between the two signals is among the classical methods used for this purpose. A generalized crosscorrelation method has also been proposed (Knapp and Carter 1976). Cross-correlation search has been used for ultrasound velocity estimation (Bonnefous and Pesque 1986; Embree and O’Brien 1990; Foster et al. 1990) along with other methods (Alam and Parker 1995; Ferrara and Algazi 1991a,b; Magnin 1987). In sonar applications, the cross-correlation is usually performed between a signal and a time-scaled version of the signal (due to Doppler shifts) in the presence of additive noise (Remley 1963); by definition, such signals are jointly nonstationary. In this case, estimates of differential Doppler and relative time-delay are found with time-companding crosscorrelators and appropriate signal filtering (Betz 1984). The effect of uncompensated time-scaling on the correlation in sonar applications has been investigated (Adams et al. 1980; Betz 1984; Betz 1985; Remley 1963).

In elastography, the precompression and postcompression signals are also jointly nonstationary. Because of the slight compression of the tissue, the location of the scatterers changes. In a one-dimensional model, only axial strain is present. If the strain is uniform, the scatterer locations are simply scaled. Consequently, the cross-correlation between precompression and postcompression echo signals deviates from the ideal autocorrelation function of the precompression (or postcompression) echo signal.

We develop an analytical expression of the cross-correlation function between the precompression and temporally stretched postcompression signals in terms of the precompression autocorrelation function. Initially, the analysis follows the work of Betz (1984) for sonar signals that are temporally scaled due to the Doppler effect. In elastography, the precompression and postcompression echo-signal are assumed to be zero-mean signals modeled by:

\[ r_1(t) = s_1(t) + n_1(t) = s(t) * p(t) + n_1(t) \]  \hspace{1cm} (1a)

\[ r_2(t) = s_2(t) + n_2(t) \]

\[ = s \left( \frac{t}{a} - t_0 \right) * p(t) + n_2(t). \]  \hspace{1cm} (1b)

If we temporally stretch the postcompression echo signal by the factor by which the scatterers were compressed, then:

\[ r_3(t) = r_2(at) = s_3(t) = s_3(t) + n_3(t) \]

\[ = s(t - t_0) * p(at) + n_3(t). \]  \hspace{1cm} (1c)
In eqns (1), \( s(t) \) is the one-dimensional scattering distribution of the elastic target, \( p(t) \) is the impulse response of the ultrasonic system, \( n_1(t) \) and \( n_2(t) \) are uncorrelated renditions of random noise and \( * \) denotes convolution. \( n_3(t) \) is a time-scaled version of \( n_2(t) \). The constant:

\[
a = 1 - \epsilon
\]

is close to unity, where \( \epsilon \) is the applied strain and in general for elastography, \( \epsilon < 0.01 \) \(<1\%\) strain) (Ophir et al. 1991). The correlation coefficient function of the pre-compression and stretched postcompression signals is given by:

\[
\rho_{13}(\tau) = \frac{C_{13}(\tau)}{\sigma_1(\tau)\sigma_3(\tau)},
\]

where \( \sigma_1(\tau) \) and \( \sigma_3(\tau) \) are the standard deviations of the corresponding indexed signals and \( C_{13}(\tau) \) is the cross-correlation function. The correlation coefficient is given by \( \rho_{13}(0) = \rho_{13} \). The cross-correlation function can be estimated by:

\[
\hat{C}_{13}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} r_1(t)r_3(t + \tau)dt,
\]

where \( T \) is the integration time and \( \hat{C}_{13}(\tau) \) denotes the estimate of \( C_{13}(\tau) \). Because of the finite integration time, the cross-correlation function estimate in eqn (4) is a random variable, and must be described statistically. The mean cross-correlation function is defined as the expectation of \( \hat{C}_{13}(\tau) \). It is interesting to note that, when the signals are jointly stationary,

\[
\lim_{T \to \infty} \hat{C}_{13}(\tau) = C_{13}(\tau).
\]

In the case of strain estimation, the signals are not jointly stationary (because displacements are depth dependent), if stretching is not applied. However, when stretching is applied, they become jointly stationary (because the depth dependence goes away). The expected value of the cross-correlation function can be calculated by considering an ensemble of the echo-signal segments, \( r_1(t) \) and \( r_3(t) \), that are uncorrelated with each other and with the noise; then, the expected (mean) cross-correlation function is given by:

\[
E\{\hat{C}_{13}(\tau)\} = \frac{1}{T} \int_{-T/2}^{T/2} E\{r_1(t)r_3(t + \tau)\}dt
= \frac{1}{T} \int_{-T/2}^{T/2} E\{s_1(t)s_3(t + \tau)\}dt,
\]

and, expressing the cross-correlation function in terms of the cross-spectrum (cross-spectral density function), we obtain:

\[
E\{\hat{C}_{13}(\tau)\} = \frac{1}{T} \int_{-T/2}^{T/2} \int_{-\infty}^{\infty} G_{13}(f)e^{j2\pi f\tau}df\,dt,
\]

where the cross-spectrum \( G_{13}(f) \) is given by:

\[
G_{13}(f) = \int_{-\infty}^{\infty} E\{s_1(t)s_3(t + \tau)\}e^{-j2\pi f\tau}d\tau.
\]

Using eqn (1c), we rewrite the above equation expressing \( s_3(t) \) in terms of its components:

\[
G_{13}(f) = E\left\{s_1(t)\right\}
\times \int_{-\infty}^{\infty} [s(t + \tau - t_0)*p(at)]e^{-j2\pi f\tau}d\tau.
\]

The convolution theorem states that the Fourier transform of the convolution of two functions is equal to the product of the Fourier transforms of the functions.

\[
G_{13}(f) = E\left\{s_1(t)\right\} \int_{-\infty}^{\infty} s(t + \tau - t_0)
\times e^{-j2\pi f\tau}d\tau \int_{-\infty}^{\infty} p(a\tau)e^{-j2\pi f\tau}d\tau.
\]

To be able to express the cross-spectrum in terms of the echo signals, we multiply and divide eqn (9) by \( \int_{-\infty}^{\infty} p(\tau)e^{-j2\pi f\tau}d\tau \), so that

\[
G_{13}(f) = E\left\{s_1(t)\right\} \int_{-\infty}^{\infty} s(t + \tau - t_0)
\times e^{-j2\pi f\tau}d\tau \int_{-\infty}^{\infty} p(\tau)e^{-j2\pi f\tau}d\tau
\times \frac{\int_{-\infty}^{\infty} p(a\tau)e^{-j2\pi f\tau}d\tau}{\int_{-\infty}^{\infty} p(\tau)e^{-j2\pi f\tau}d\tau}.
\]

The system impulse response \( p(t) \) is deterministic. Thus, the ratio of spectra in eqn (10) can be taken out of the expectation operator. The time shift property of Fourier transforms is also used for the scatterer function.
\[
G_{13}(f) = E \left\{ s_1(t) e^{-j2\pi f_0} \int_{-\infty}^{\infty} s(t + \tau) \right. \\
\left. \times e^{-j2\pi f \tau} d\tau \int_{-\infty}^{\infty} p(\tau) e^{-j2\pi f \tau} d\tau \right\}
\]

\[
= \int_{-\infty}^{\infty} \frac{p(\alpha \tau) e^{-j2\pi f \tau}}{p(\tau) e^{-j2\pi f \tau}}. \quad (11)
\]

Since the product of the Fourier transforms of two functions can be expressed as the Fourier transform of the convolution of the two functions, it follows that:

\[
G_{13}(f) = E \left\{ s_1(t) e^{-j2\pi f_0} \int_{-\infty}^{\infty} s(t + \tau) * p(\tau) e^{-j2\pi f \tau} d\tau \right\}
\]

\[
= \int_{-\infty}^{\infty} \frac{p(\alpha \tau) e^{-j2\pi f \tau}}{p(\tau) e^{-j2\pi f \tau}}. \quad (12)
\]

and thus,

\[
G_{13}(f) = E \left\{ s_1(t) \int_{-\infty}^{\infty} s_1(t + \tau) e^{-j2\pi f \tau} d\tau \right\}
\]

\[
= \frac{e^{-(f_0 + f_0^2)/2\sigma^2}}{e^{-(f_0 - f_0^2)/2\sigma^2}} e^{-j2\pi f_0}. \quad (13)
\]

Rearranging the integration and the expectation operations,

\[
G_{13}(f) = \int_{-\infty}^{\infty} E \left\{ s_1(t) s_1(t + \tau) \right\} e^{-j2\pi f \tau} d\tau
\]

\[
= \frac{e^{-(f_0 + f_0^2)/2\sigma^2}}{e^{-(f_0 - f_0^2)/2\sigma^2}} e^{-j2\pi f_0}. \quad (14)
\]

From this it follows that:

\[
G_{13}(f) = G_{11}(f) \frac{e^{-(f_0 - f_0^2)/2\sigma^2}}{e^{-(f_0 + f_0^2)/2\sigma^2}} e^{-j2\pi f_0}. \quad (15)
\]

where \( G_{11}(f) \) is the autospectrum (autospectral density function) for the precompression A-line.

Now, by using the time scaling properties of Fourier transforms,

\[
P \left( \frac{f}{a} \right) = \frac{G_{13}(f)}{a P(f)} e^{-j2\pi f_0} \quad (16)
\]

where \( P(f) = \int_{-\infty}^{\infty} p(\tau) e^{-j2\pi f \tau} d\tau. \)

It had been demonstrated that the cross-spectrum of the signals in eqns (1a) and (1b) is nonstationary in nature, since the cross-spectrum is a function of the autospectrum and a function of time (Césedes 1993). However, the cross-spectrum of the signals in eqns (1a) and (1c) is not dependent on time and thus is stationary, as long as the signals in eqn (1a) and (1b) are stationary.

If we assume a zero-mean Gaussian impulse response of the ultrasound system,

\[
p(t) = 2\sqrt{2\pi} \sigma e^{-2(\sigma^2t)^2} \sin(2\pi f_0 t) \Rightarrow P(f) = -j \left\{ e^{-(f - f_0^2)/2\sigma^2} - e^{-(f + f_0^2)/2\sigma^2} \right\},
\]

where \( \Rightarrow \) denotes a Fourier transform relationship.

Then,

\[
G_{13}(f) = G_{11}(f) \frac{e^{-(f + f_0^2)/2\sigma^2} - e^{-(f - f_0^2)/2\sigma^2}}{a \left\{ e^{-((f + f_0^2)/2\sigma^2)} - e^{-(f - f_0^2)/2\sigma^2} \right\} e^{-j2\pi f_0}}
\]

\[
= \frac{1}{a} G_{11}(f) e^{-(f - f_0^2)/2\sigma^2} \left\{ e^{-(f + f_0^2)/2\sigma^2} - e^{-(f - f_0^2)/2\sigma^2} \right\} e^{-j2\pi f_0}
\]

\[
= \frac{1}{a} G_{11}(f) e^{-(f - f_0^2)/2\sigma^2} \left\{ e^{(f_0^2)/2\sigma^2} - e^{-(f_0^2)/2\sigma^2} \right\} e^{-j2\pi f_0}
\]

\[
= \frac{1}{a} G_{11}(f) e^{-(f - f_0^2)/2\sigma^2} \left\{ e^{f_0^2/2\sigma^2} - e^{-f_0^2/2\sigma^2} \right\} e^{-j2\pi f_0}
\]

\[
= \frac{1}{a} G_{11}(f) e^{-(f - f_0^2)/2\sigma^2} \sinh(f_0/\sigma^2) \sinh(\sigma^2/2) \sinh(f_0/\sigma^2) e^{-j2\pi f_0}. \quad (17)
\]

As elastography primarily deals with small compressions, \( a \) is very close to unity. Thus, in the limit:

\[
\lim_{a \to 1} G_{13}(f) = \lim_{a \to 1} G_{11}(f) e^{-(1-a^2)^2/2(\sigma a)^2} \sinh(f_0/\sigma^2) \sinh(\sigma^2/2) \sinh(f_0/\sigma^2) \sinh(f_0/\sigma^2) e^{-j2\pi f_0}. \quad (18)
\]

Replacing eqn (17) in eqn (6) will yield:
\[
E\{\hat{C}_{13}(\tau)\} = \frac{1}{T} \int_{-T/2}^{T/2} \int_{-\infty}^{\infty} G_{11}(f) e^{-\left(1-a^2f^2\gamma_2(\alpha\sigma)^2\right)} \frac{\sinh(\beta f_0/a\sigma^2)}{\sinh(\beta f_0/\sigma^2)} \times e^{-j2\pi f_0} e^{j2\pi f \tau} df \, dt. \tag{19}
\]

The inner integral is not dependent on time. Thus, eqn (19) simplifies to:

\[
E\{\hat{C}_{13}(\tau)\} = \int_{-\infty}^{\infty} G_{11}(f) e^{-\left(1-a^2f^2\gamma_2(\alpha\sigma)^2\right)} \frac{\sinh(\beta f_0/a\sigma^2)}{\sinh(\beta f_0/\sigma^2)} \times e^{-j2\pi f_0} e^{j2\pi f \tau} df, \tag{20}
\]

and, finally, the product in the frequency domain can be expressed as a convolution operation in the time domain. That further simplifies the eqn (20) to

\[
E\{\hat{C}_{13}(\tau)\} = R_{11}(\tau) * h(\tau) * \delta(\tau - t_0), \tag{21}
\]

where \(R_{11}(\tau)\) is the autocorrelation function for the signal in eqn (1a) and:

\[
h(\tau) = F^{-1}\left\{e^{-\left(1-a^2f^2\gamma_2(\alpha\sigma)^2\right)} \frac{\sinh(\beta f_0/a\sigma^2)}{\sinh(\beta f_0/\sigma^2)} \right\}. \tag{22}
\]

For the small compression cases where \(a \to 1\), eqn (22) can be simplified in the following form,

\[
\lim_{a \to 1} h(\tau) = \lim_{a \to 1} F^{-1}\left\{e^{-\left(1-a^2f^2\gamma_2(\alpha\sigma)^2\right)} \frac{\sinh(\beta f_0/a\sigma^2)}{\sinh(\beta f_0/\sigma^2)} \right\} \tag{23}
\]

\[
= \lim_{a \to 1} F^{-1}\{1\} = \delta(\tau). \tag{23}
\]

Inspection of eqns (16), (17), and (21) shows that the effect of tissue compression followed by the temporal stretching of the echo signal is equivalent to passing the ideal autocorrelation through a low-pass filter of bandwidth defined by the strain, carrier frequency, and the pulse width; the impulse response of the filter is given by eqn (22). For small compression, which is generally the case in elastography, the limiting cases in eqns (18) and (23) show that temporal stretching almost completely offsets the decorrelation caused by the tissue compression.

### Inverse filtering

In the preceding section, we have shown that the cross-correlation function \(C_{13}\) can be made to closely resemble the autocorrelation function of the postcompression signal by making the strains very small. However, the precompression signal and the stretched signal in eqns (1a) and (1c), respectively, still are different due to the difference in the original and stretched system impulse responses \(p(t)\) and \(p(at)\) embedded in them. Thus, we can further improve the performance of elastography by incorporating an inverse filter that is to be applied to the stretched signal prior to the cross-correlation step.

From eqn (1),

\[
r_1(t) = s_1(t) + n_1(t) = s(t)*p(t) + n_1(t),
\]

\[
r_2(t) = s_2(t) + n_2(t) = s\left(\frac{t}{a} - t_0\right)*p(t) + n_2(t)
\]

and

\[
r_3(t) = r_2(t) = s_3(t) + n_3(t)
\]

\[
= s(t - t_0)*p(at) + n_3(t).
\]

Next, we evaluate the Fourier transform of \(r_1(t)\) and \(r_3(t)\),

\[
R_1(f) = S_1(f) + N_1(f) = S(f)P(f) + N_1(f), \tag{24}
\]

\[
R_3(f) = S_3(f) + N_3(f) = \frac{1}{a} S(f)P\left(\frac{f}{a}\right) e^{-j2\pi f_0} + N_3(f). \tag{25}
\]

Clearly, under ideal circumstances (no noise, no decorrelation or attenuation, etc.), an equalization filter can be applied to the stretched signal to perfectly reconstruct the original postcompression echo signal (Céspedes 1995).

Under ideal conditions,

\[
R_3(f) = \frac{1}{a} S(f)P\left(\frac{f}{a}\right) e^{-j2\pi f_0}. \tag{26}
\]

Let us define a filter \(H(f) = \frac{P(f)}{P(f/a)}\) for equalizing \(R_3(f):\)

\[
R_3(f)H(f) = \frac{1}{a} S(f)P\left(\frac{f}{a}\right) e^{-j2\pi f_0} \frac{P(f)}{P(f/a)} = \frac{1}{a} S(f)P(f) e^{-j2\pi f_0} = \frac{1}{a} R_1(f) e^{-j2\pi f_0}. \tag{27}
\]

Thus, under ideal circumstances, it is possible to reconstruct the stretched signal perfectly as long as the system transfer function \(P(f)\) is known and it does
not have any zeroes in the frequency range where the filter would be applied. Note that, for a Gaussian impulse response, the filter is a high frequency emphasis filter. Now, with noise present, if the equalization filter is applied,

\[
R_3(f) \frac{P(f)}{P(f/a)} = \{S_3(f) + N_3(f)\} \frac{P(f)}{P(f/a)}
\]

\[
= \left\{ \frac{1}{a} S(f) P \left( \frac{f}{a} \right) e^{-\frac{f}{2\pi f_0}} + N_3(f) \right\} \frac{P(f)}{P(f/a)}
\]

\[
= \frac{1}{a} S(f) P \left( \frac{f}{a} \right) e^{-\frac{f}{2\pi f_0}} + N_3(f) \frac{P(f)}{P(f/a)}
\]

\[
= \frac{1}{a} S(f) e^{-\frac{f}{2\pi f_0}} + N_3(f) \frac{P(f)}{P(f/a)}. \tag{28}
\]

By looking at the above equation, we see that care should be taken when applying this equalization filter on noisy signals. The equalization filter should be applied only in the frequency range where the SNR is greater than one. Otherwise, the noise will be amplified over the signal at the frequencies where the noise power is dominant. Thus, the equalization filter should be designed as follows:

\[
H(f) = \begin{cases} 
P(f) / P(f/a), & S_3(f) \geq N_3(f) \text{ and } P(f/a) \neq 0 \\
0, & \text{otherwise.} \end{cases} \tag{29}
\]

**Inverse filtering of the crosscorrelation functions**

From eqns (27) and (28), we see that we can possibly get even better performance from the time delay estimator using the inverse filter if the local strain and the system impulse response are known. Equivalently, from eqn (16), it appears that the cross-correlation function can also be passed through a filter \( Q(f) \) such that:

\[
Q(f) = \frac{aP(f)}{P(f/a)}, \text{ and} \tag{30}
\]

\[
G_{13}(f) Q(f) = G_{11}(f) \frac{P(f/a)}{aP(f)} e^{-\frac{f}{2\pi f_0}} \frac{aP(f)}{P(f/a)}
\]

\[
= G_{11}(f) e^{-\frac{f}{2\pi f_0}}. \tag{31}
\]

**SIMULATION AND PHANTOM EXPERIMENT**

We performed a one-dimensional simulation to verify the theory. We simulated a line of 245 uniformly spaced random amplitude (Gaussian distributed) scatterers within a transducer beam. Ten scatterers per wavelength were simulated. For the simulated roundtrip transfer function, the center frequency was 5 MHz and the 10-dB fractional bandwidth was 85\%. The radiofrequency (RF) A-line was calculated by convolving the scatterer profile with the impulse response of the system. The RF A-lines were sampled at 5 GHz to simulate tissue compression without any interpolation. The scatterer spacing was then uniformly reduced to simulate tissue compression and a delay of 0.1 \( \mu s \) was also applied at \( t = 0 \). The RF A-line was computed again. After the postcompression RF A-line was computed, it was temporally stretched by the factor by which the scatterer spacing had been reduced. We had two separate cases: (1) no additive noise was added to the RF A-lines; and (2) additive noise was added to the RF lines such that the SNR was 10 dB. To test the theory, we computed the autocorrelation function for the precompression A-line and the cross-correlation function between the precompression and the postcompression A-lines, and also between the precompression A-line and the stretched postcompression A-line. A window size of 2.4 \( \mu s \) was used for the correlation operations. These correlation functions were averaged over 16 realizations. Fourier transformations of the correlations functions were taken to get the auto- and cross-spectra. Then, we used eqn (16) to theoretically evaluate the cross-spectrum between the precompression A-line and the stretched postcompression A-line from the autospectrum.

We also performed a homogeneous phantom experiment to test the validity of eqn (16). The data were collected using a Diasonics Spectra scanner with a 5-MHz transducer. A gel-based homogeneous tissue-mimicking phantom, constructed by Dr. Tim Hall of the Department of Radiology at the University of Kansas Medical Center, was used. First, the precompression RF image was collected. Then we applied a known compression and recorded the postcompression RF image. A spline-based interpolation technique was used to stretch the postcompression RF A-lines. For the correlation operations, only segments from the focal region and far field were used, to avoid the near-field effects. The window size for the correlation operations was about 6 \( \mu s \). The correlation functions were averaged for 16 A-lines around the lateral center of the RF image. Choosing these RF lines helps to reduce the effect of the non-axial decorrelations. Finally, the auto- and cross-spectra were evaluated in a manner similar to that used on the simulated data.

**RESULTS AND DISCUSSION**

The results for the one-dimensional simulation described in the previous section are shown in Figs. 1–
6. There was no additive noise for the cases shown in Figs. 1–4. In Figs. 5 and 6, the SNR was 10 dB. In Figs. 1 and 2, correlation functions are plotted for applied strains of 1% and 5%, respectively. For the lower strain case in Fig. 1, the cross-correlation functions for the stretched and unstretched cases are quite close, but the former is higher and closer to the autocorrelation function. The autocorrelation function maximizes at zero as expected, the cross-correlation function between the precompression and stretched postcompression RF A-lines maximizes at a delay of slightly >0.1 μs (due to stretching; 0.1 μs delay was applied at t = 0, in addition to the compression) and the correlation function between the precompression and postcompression RF A-lines maximizes at slightly <0.1 μs, since the compression modified the mean shift between these windowed waveforms. It should be emphasized that the small improvement in correlation due to stretching can be misleading, because even a small increase in the correlation can significantly improve the estimator performance, especially at lower strains (Céspedes et al. 1997). At the higher strain case in Fig. 2, the correlation function with stretching still has a distinct peak similar in height to the autocorrelation function (it can be thought of as the cross-correlation function with zero strain), whereas the cross-correlation function for the unstretched case has severely degenerated and no longer has a distinct peak. Thus, temporal stretching improves the estimator performance at both low and high strains.

Figures 3 and 4 show the corresponding spectral behavior for 1% and 5% strains, respectively. At 1% strain, the cross-spectrum for the unstretched case is slightly lower than the autospectrum. Once stretching is applied, the difference becomes negligible. For the case of 5% strain, however, the cross-spectrum for the unstretched case is significantly lower than the autospectrum. Stretching reduces the difference significantly. The computed cross-spectrum between the precompression signal and the stretched postcompression signal $G_{13}(f)$ and the theoretically evaluated $G_{13}(f)$ are virtually identical in both Figs. 3 and 4. The difference could have been reduced even more if the window size for the cross-correlation operation were further increased because then the cross-correlation operation would be closer to its classical definition, having infinite window width. No error bars are shown here because the spectra are derived from non-normalized correlation functions and, hence, their variances could be quite large and not very meaningful in validating the eqn (16).

Figures 5 and 6 are similar to Figs. 3 and 4. The only difference is that noise was added and the SNR was 10 dB. The computed cross-spectrum for the stretched echo $G_{13}(f)$ and the theoretically evaluated cross-spectrum from the autospectrum in both figures are very close here also, validating eqn (16), even in the presence of noise.

Figures 7–9 shows the results for the homogeneous phantom experiment described in the previous section. Figure 7 shows the various auto- and cross-spectra for 1% strain. At 1% strain, the cross-spectrum between the precompression and postcompression signals is slightly lower than the autospectrum of the precompression signal. However, once stretching is applied, the cross-spectrum is pretty close to the autospectrum. The computed cross-spectrum between the precompression signal and the stretched postcompression signal $G_{13}(f)$ is very close to the theoretically
Fig. 3. One-dimensional simulation. Average spectra for 1% strain: $G_{11}$ — autospectrum of precompression A-line; $G_{12}$ — cross-spectrum of precompression and postcompression A-lines; $G_{13}$ — cross-spectrum of precompression and stretched postcompression A-lines; and $G_{13\text{-theor.}}$ — theoretically evaluated cross-spectrum of precompression and stretched postcompression A-lines from $G_{11}$ using eqn (16).

evaluated $G_{13}(f)$. Figures 8 and 9 show the various auto- and cross-spectra for 2% strain. In Fig. 8, at 2% strain, the cross-spectrum between the precompression and postcompression signals is significantly lower than the auto-spectrum of the precompression signal. However, once stretching is applied, the cross-spectrum is much closer to the autospectrum. Figure 9 shows only the computed cross-spectrum between the precompression signal and the stretched postcompression signal $G_{13}(f)$ and the theoretically evaluated $G_{13}(f)$. They are quite close, further validating eqn (16). However, the computed one is slightly lower than the theoreti-

Fig. 4. One-dimensional simulation. Average spectra for 5% strain: $G_{11}$ — autospectrum of precompression A-line; $G_{12}$ — cross-spectrum of precompression and postcompression A-lines; $G_{13}$ — cross-spectrum of precompression and stretched postcompression A-lines; and $G_{13\text{-theor.}}$ — theoretically evaluated cross-spectrum of precompression and stretched postcompression A-lines from $G_{11}$ using eqn (16).

Fig. 5. One-dimensional simulation. Average spectra for 1% strain; SNR = 10 dB: $G_{11}$ — autospectrum of precompression A-line; $G_{12}$ — cross-spectrum of precompression and postcompression A-lines; $G_{13}$ — cross-spectrum of precompression and stretched postcompression A-lines; and $G_{13\text{-theor.}}$ — theoretically evaluated cross-spectrum of precompression and stretched postcompression A-lines from $G_{11}$ using eqn (16).

Fig. 6. One-dimensional simulation. Average spectra for 5% strain; SNR = 10 dB: $G_{11}$ — autospectrum of precompression A-line; $G_{12}$ — cross-spectrum of precompression and postcompression A-lines; $G_{13}$ — cross-spectrum of precompression and stretched postcompression A-lines; and $G_{13\text{-theor.}}$ — theoretically evaluated cross-spectrum of precompression and stretched postcompression A-lines from $G_{11}$ using eqn (16).
Fig. 7. Phantom experiment. Average spectra for 1% strain: $G_{11}$—autospectrum of precompression A-line; $G_{12}$—cross-spectrum of precompression and postcompression A-lines; $G_{13}$—cross-spectrum of precompression and stretched postcompression A-lines; and $G_{13}$-theor.—theoretically evaluated cross-spectrum of precompression and stretched postcompression A-lines from $G_{11}$ using eqn (16).

Fig. 8. Phantom experiment. Average spectra for 2% strain: $G_{11}$—autospectrum of precompression A-line; $G_{12}$—cross-spectrum of precompression and postcompression A-lines; $G_{13}$—cross-spectrum of precompression and stretched postcompression A-lines; and $G_{13}$-theor.—theoretically evaluated cross-spectrum of precompression and stretched postcompression A-lines from $G_{11}$ using eqn (16).

Fig. 9. Phantom experiment. Average spectra for 2% strain: $G_{13}$—cross-spectrum of precompression and stretched postcompression RF A-lines; and $G_{13}$-theor.—cross-spectrum of precompression and stretched RF A-lines from the autospectrum of the precompression RF A-line using eqn (16).

$G_{11}(f)$ and $G_{12}(f)$ was 0.9624. The same between $G_{11}(f)$ and $G_{13}(f)$ was 0.9926, demonstrating that stretching does indeed improve the correlation. Finally, the degree of correlatedness between theoretically evaluated $G_{13}(f)$ and computed $G_{13}(f)$ is 0.9931, thus corroborating the theory. For the 2% strain experiment, the degree of correlatednesses, in the same order, are: 0.8356, 0.9827, and 0.9837. So, stretching significantly improves the correlation and the theory is strongly validated. The drop from 0.9931 to 0.9837 as the strain increases from 1% to 2% can be attributed to the other decorrelation effects not accounted for in the model, such as lateral and elevational decorrelations.

Since the elastographic signal-to-noise ratio ($\text{SNR}_e$), is nonlinearly related to the maximum value of the cross-correlation coefficient function (Céspedes et al. 1997), the above-mentioned figures may not provide a complete picture of the elastographic performance enhancement when temporal stretching is applied. The performance of elastography can be measured either from $\text{SNR}_e$, or equivalently, since they are related, from the maximum value of the cross-correlation coefficient function. It had been determined that the maximum value of the cross-correlation coefficient function should be more than 0.93 for acceptable $\text{SNR}_e$ (Varghese and Ophir 1997). Below this value, the $\text{SNR}_e$ drops very sharply. Figure 10 shows the plot of the mean value of the cross-correlation coefficient function maximum (averaged over 32 realizations) as a function of increasing strain in a one-dimensional simulation. The mean value of the cross-correlation coefficient function maximum falls below the threshold of 0.93 at slightly above 2% strain, if no stretching was applied. However, with stretching, the
mean value for the correlation coefficient function maximum remains slightly higher than the threshold at 0.93, even at a strain of 10%. At any applied strain, the maximum cross-correlation coefficient function is higher when stretching is applied, which implies that the SNR of the elastogram would also be significantly higher.

Note that the tissues examined by elastography are not expected to be elastically homogeneous. Thus, the strain would also be spatially varying. Let us consider an example where the total applied strain is 1%, but due to the inhomogeneities the local strain in the tissue varies from 0.1%–5%. If an RF segment from an area that has a strain of 3% is temporally stretched by 1%, then the situation is similar to the case with 2% strain with no stretching. Thus, to get the best performance out of stretching, the postcompression echo has to be stretched by the appropriate amount. This can be done iteratively, where the temporal stretching factor in any given segment would be modified until a maximum for the peak value for the cross-correlation coefficient function is found. Alternatively, one can look very closely at the signals within each signal segment and apply appropriate temporal stretching. Note also that the temporal stretching factor itself may be used as a good direct estimator of local tissue strain since its use closely reconstructs the original signal (Alam and Ophir 1997).

CONCLUSIONS

We have presented a theoretical model for the cross-correlation function of precompression and temporally stretched postcompression echo-signals expressed as a function of the autocorrelation function of the precompression echo-signal. The effect of tissue compression followed by temporal signal stretching was shown to be equivalent to low-pass filtering of the precompression autocorrelation function. The one-dimensional simulation and the homogeneous phantom results show good agreement with the theory. At either low or high strains, appropriate temporal stretching can significantly improve the elastographic performance.

Echo-signal decorrelation is one of the major limiting factors in strain estimation and imaging. The theory and simulation results demonstrate that the axial decorrelation can almost entirely be compensated for, as long as the applied strain is small. Decorrelation due to lateral and elevational strain is assumed to be much smaller than their axial counterpart, and has not been addressed in this article.

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Fig. 10. Illustration of the improvement of performance when stretching is applied using the maximum value of the cross-correlation coefficient function as an indicator.


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