Three-Dimensional Tissue Motion and Its Effect on Image Noise in Elastography

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Abstract—In elastography both high correlation coefficient between pre- and post-compression RF signals and high applied strain are required to achieve the best quality in elastograms. Because the elastogram is computed using a 1-D cross-correlation technique applied to a 1-D ultrasound signal, it is assumed that tissue motion occurs only within the axis of compression (axis of the acoustic wave propagation), or at least that the scatterers remain within the acoustic beam during tissue motion. In practice, soft tissues are incompressible and, therefore, the lateral and elevational (out-of-plane) tissue strains are 50% of the applied strain. Therefore, tissue scatterers may move across the beam due to the applied compression. In this paper we address the degradation of the elastographic quality due to the lateral and elevational motion of the scatterers in uniformly elastic media. A full 3-D model predicting the correlation coefficient as measured using 1-D cross-correlations is proposed. It is shown that the signal-to-noise ratio in elastograms (SNR̄) is nonstationary, and that it depends on the beamwidth and on the applied strain. In order to achieve a higher stationary SNR̄, it is proposed to confine the tissue in the lateral direction. Phantom experiments are used to corroborate the theoretical developments.

I. INTRODUCTION

Pathological changes are generally correlated with changes in tissue stiffness. For example, many cancers, such as scirrhous carcinoma of the breast, appear as extremely hard nodules [1]. On this basis, elastography was recently proposed as a new medical imaging modality to display stiffness changes in soft tissue [2]. More recently, the potential of the clinical application of such technique to the diagnosis of breast cancer has been demonstrated [3].

As is the case for any imaging modality, elastography has its unique characteristics such as spatial resolution, signal-to-noise ratio (SNR), contrast-transfer efficiency and artifacts. Some of these characteristics have been studied [4]–[16] and others are under current investigation. In order to represent these characteristics, elastography has recently been described as a cascade of two distinct processes [17]. The first process involves the mapping of the distribution of local elastic moduli in the target into a distribution of local longitudinal strains. This process is governed by the theory of elasticity as applied to a particular experimental setup under some specific boundary conditions and some assumptions. Because this process involves errors due to the simplified mechanical model used, artifacts such as Target-Hardening, stress concentrations and limited contrast-transfer efficiency are usually encountered [4], [8], [17]. To reduce these artifacts, elastography should be considered within the framework of inverse problem solving [6], [18], [19]. The second process involves the production of the strain image (elastogram) from ultrasonically estimated values of local strains. Here the limitations of the ultrasound system (such as time-bandwidth product, center frequency, beamwidth, and sonographic SNR), as well as the algorithms used to process the signals cause additional corruption of the data by introducing constraints in the attainable elastographic SNR, resolution, strain dynamic range, and sensitivity (smallest measured tissue strain). Notice that in practice the data used to reconstruct the tissue stiffness distribution are those produced by the second process (estimated tissue strains and displacements). Therefore, a complete understanding of these two inter-dependent processes is the key for further improvement of image quality in elastography. In this paper we focus on the second process by investigating the effect of the ultrasonic beam on elastogram quality.

In elastography, tissue axial displacements induced by an external compression are measured using a 1-D cross-correlation technique applied to the RF echo signals. More specifically, the tissue displacement field is computed using cross-correlation applied to a pair of segments taken from pre- and post-compression RF signals [2]. In this computation, in each small segment the tissue motion is assumed to be constant and to occur only in the axial direction [31]. The same assumption is also made by O’Donnell et al. [32] to compute the tissue strain using the Fourier speckle tracking technique.

The measure of performance of the cross-correlation based delay estimator is well established in many fields [20]. Most of these models have been developed for pure time delay estimation, except a few studies where the problem of a moving target was particularly addressed [20]. In elastography we deal with an additional signal decorrelation due to physical tissue distortion. Recently, some of the time delay noise models have been adapted to the strain estimator in elastography by including the effects of signal distortions [10], [12]–[16], [25]. These are 1-D models based on the assumption that tissue deforms uniformly and uniaxially under an external compression. Indeed, the tissue is mechanically modeled as a spring (perfectly compressible) and acoustically as a 1-D random distribution of scatterers.
The measure of performance of elastography proposed by Varghese and Ophir [10] uses a theoretical framework known as the strain filter. The strain filter (SF) predicts the signal-to-noise ratio in the elastogram (SNRe), its dynamic range (DR), and sensitivity for a given set of known imaging system parameters (such as resolution) as well as the processing parameters.

For the theoretical SF the measured tissue strain is assumed to be equal to the applied strain. The standard deviation is predicted using a closed form expression of the lower bound of variance of the time delay as measured from two noisy bandpass signals [21], [22]. In elastography the post-compression RF echo signal is not a delayed version of its pre-compression signal. Indeed, due to the applied strain the tissue scatterers move relative to each other, thus the resulting pre- and post-compression acoustic speckle patterns will be different. This difference is known as speckle decorrelation [24], [25]. Therefore, to use the models developed for pure time delay estimation, the post-compression signal is modeled as a pure delayed version of its corresponding pre-compression signal plus an additional noise term. This noise is known as the decorrelation noise which results from the motion of tissue scatterers relative to each other. It is measured from the correlation coefficient between the pre- and post-compression signals. Moreover, the correlation coefficient may be converted to a sonographic SNRs term using the following relation [25]:

\[
\text{SNR}_\text{e} = \frac{\rho}{1 - \rho}. \tag{1}
\]

Using the SF concept, it was shown that the elastographic SNRe is highly dependent on the correlation coefficient [10]. Indeed, it was demonstrated that a slight decrease of this correlation coefficient from a value of 1 results in a dramatic decrease of the SNRe, due to the highly nonlinear nature of (1) for \(\rho\) close to 1. Therefore, it is important to evaluate the realistic level of signal decorrelation due to all sources in order to be able to properly derate the strain filter [10].

The approach proposed by Bilgen and Insana [14], [15] to measure the elastographic quality is quite similar to the approach proposed by Varghese and Ophir [10]. In Bilgen and Insana [14], [15], closed form expressions for the variance of the estimated displacement and strain were derived. These expressions are a function of the imaging system parameter and the signal processing parameters, and are valid only when there is no ambiguity in the correlation peak detection. The estimated strain variance is then used to predict a noise-figure measure defined as the ratio of the elastogram SNRe to the sonographic SNR. When plotted as a function of applied strain, the elastogram noise-figure is similar to a strain filter.

In elastography, tissue scatterers undergo a complex motion pattern depending on the boundary conditions and elastic properties of the tissue. Moreover, because soft tissues are incompressible [26], [27], the motion of their scatterers occurs in a 3-D space. Therefore, the assumption of the uniaxial motion of tissue scatterers during compression is not valid and may result in an overestimation of elastogram performance, as predicted by Varghese and Ophir [10] using the SF and by Bilgen and Insana [15].

In this paper, we take these effects into account in order to predict more realistic elastographic performance. The paper is organized as follows: in Section II and in the Appendix the correlation coefficient measured using pre- and post-compression A-lines is theoretically predicted for tissue undergoing a 3-D motion. To corroborate the theoretical development, experimental results using a gel phantom are presented in Section III. To improve the elastographic performance, in Section IV, we propose to confine the tissue in the lateral direction. Section V summarizes the contribution of the paper.

II. THEORY

In order to account for scatterer motion in and out of the ultrasonic beam during tissue compression, it is necessary to use a complete 3-D model of the echographic image formation process. The 3-D convolutional model we use in this paper is presented for a system scanning linearly in the \((x, y)\) plane, using the coordinate system shown in Fig. 1. In this model the 3-D RF echo \(i(x, y, z)\) is given by:

\[
i(x, y, z) = \int \int h(x - x', y - y', z - z') \times t(x', y', z') dx' dy' dz' \tag{2}
\]

where \(h(x, y, z)\) is the system's 3-D point spread function (PSF), \(t(x, y, z)\) is the tissue scattering function and \(x, y, \text{ and } z\) define the spatial system impulse response position in the elevational, lateral (scanning axis), and axial
directions (ultrasound propagation axis), respectively. For this model it is assumed that the system is linear and has a shift invariant impulse response which holds within a small volume of interest \((5 \times 5 \times 5 \text{ mm}^3)\) at the focus \([28]\). Moreover, to simplify the analysis, the echo signal is assumed to be free of acquisition noise (electronic, quantization, etc.). In elastography, strain images are computed by processing pairs of 1-D RF echo lines. From (2), assuming the ultrasound beam is located at \((x, y) = (0, 0)\), we can write the RF line \(i(0,0,z)\) as:

\[
i(0,0,z) = i_1(z) = \int \int \int h(-x', -y', z - z') t(x', y', z') dx' dy' dz'.
\]  

(3)

Assuming that the PSF is separable, an assumption that holds in the focus \([29], [30]\), we can write \(h(x, y, z)\) as:

\[
h(x, y, z) = h_x(x)h_y(y)h_z(z).
\]  

(4)

Using this property of separability, (3) may be written as:

\[
i_1(z) = \int h_z(z - z') t_1(z') dz'.
\]  

(5)

where the new tissue scattering function is given by:

\[
t_1(z') = \int t(x', y', z') h_x(-x') h_y(-y') dx' dy'.
\]  

(6)

In elastography, (3) models the pre-compression RF A-line. The post-compression RF A-line is given by:

\[
i_2(z) = \int \int \int h(-x', -y', z - z') t_2(x', y', z') dx' dy' dz'.
\]  

(7)

where \(t_c(x', y', z')\) is the compressed tissue scattering function.

For the next development we assume that the function \(h_x(x)\) and \(h_y(y)\) are both symmetric. Under the assumption that the tissue is homogeneous, isotropic, and compressed with a compressor larger than the target with a perfect slip boundary, all the components of the tissue displacement field are 1-D functions given by:

\[
\begin{align*}
u(x) &= -a y, \quad -L_x \leq x \leq L_x \\
v(y) &= -a x, \quad -L_y \leq y \leq L_y \\
w(z) &= a z, \quad 0 \leq z \leq L_z
\end{align*}
\]

(8)

where \(u(x), v(y),\) and \(w(z)\) represent the displacement of the tissue in the elevational, lateral, and axial direction, respectively, \(a\) is the applied axial strain (tissue compression), \(\nu\) is the tissue Poisson’s ratio (equal 0.5 for an incompressible tissue) and \(L_x, L_y,\) and \(L_z\) represent the size of the target. In this case and for a small applied strain \((a \ll 1)\), we can show that the compressed tissue scattering function is given by:

\[
t_c(x, y, z) = t(x - u(x), y - v(y), z - w(z)).
\]  

(9)

Thus the post-compression RF A-line is given by:

\[
i_2(z) = \int h_z(z - z') t_{2c}(z') dz' = \int t(x', y', z' - w(z')) x h_x(x') h_y(y') dx' dy'.
\]  

(10)

\[
t_{2c}(z') = \int t(x', y', z' - w(z')) x h_x(x') h_y(y') dx' dy' = \int t(x', y', z') x h_x(x') h_y(y') dx' dy'.
\]  

(11)

Within the ultrasonic beam we assume that the displacement function \(u(x)\) and \(v(y)\) are constant. In practice, this assumption is valid because the applied strain is small and the beam is relatively narrow. Therefore, simple translation of the scatterers is equivalent to the same sign reversed shift of the system impulse response, and (11) can be written as:

\[
t_{2c}(z') = \int t(x', y', z' - w(z')) x h_x(x' + u(x_0)) h_y(y' + v(y_0)) dx' dy'.
\]  

(12)

where \(x_0\) and \(y_0\) are the elevational and lateral position of the ultrasound beam relative to the target (tissue) coordinate system.

For a better prediction of elastographic performance using the concept of the strain filter (derived for lateral and elevational strain components), an effective correlation coefficient must be used. The expression of such correlation coefficient follows.

The correlation coefficient between the pre- and post-compression A-lines is given by:

\[
C_{12}(z_1, z_2) = E[i_1(z_1) i_2^*(z_2)]
\]

(13)

where \(z_1, z_2\) are two separate depths, \(E[\cdot]\) denotes the expected value and '*' denotes the complex conjugate. After tissue motion compensation \([24], [33]\), we can write (13) as

\[
C_{12}(z_1, z_2) = E[(h_z(z_1) \otimes t_1(z_1))(h_z'(z_2) \otimes t_2(z_2))^*]
\]

(14)

where \(h_z'(z_2)\) is the system axial impulse response after compensation for the tissue motion in the axial direction (i.e., axial stretching of the pre-compression RF echo line). The quantity \(t_2(z_2)\) is the new scattering function obtained after motion compensation and is given by:

\[
t_2(z_2) = \int t(x, y, z_2) h_x(x + u(x_0)) h_y(y + v(y_0)) dx dy.
\]

(15)

The convolution terms in (14) can be written as:

\[
h_z(z_1) \otimes t_1(z_1) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H_z(\omega_1) T_1(\omega_1) e^{j\omega_1 z_1} d\omega_1
\]

(16a)

\[
h_z'(z_2) \otimes t_2(z_2) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H_z'(\omega_2) T_2(\omega_2) e^{j\omega_2 z_2} d\omega_2.
\]

(16b)
Therefore, using (16a) and (16b), (14) can be written as:

\[
C_{12}(z_1, z_2) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H_z(\omega_1)H_z^*(\omega_2)E[T_1(\omega_1)T_2^*(\omega_2)] \\
\times e^{i(\omega_1 z_1 - \omega_2 z_2)} \, d\omega_1 \, d\omega_2
\]  

(17)

where the expectation is taken only over the Fourier transform of the tissue scattering functions which are modeled as white Gaussian random processes. We can show that such expectation term in (17) is given by (see the Appendix):

\[
E[T_1(\omega_1)T_2^*(\omega_2)] = S^2 \cdot C_{xx}(u(x_0)) \cdot C_{yy}(v(y_0)) \delta(\omega_1 - \omega_2)
\]  

(18)

where \(S^2\) is the variance of the white Gaussian random process modeling the tissue 3-D scattering profile and the covariance functions are given by:

\[
C_{xx}(u(x_0)) = \int h_x(x)h_x(x + u(x_0)) \, dx
\]  

(19a)

\[
C_{yy}(v(y_0)) = \int h_y(y)h_y(y + v(y_0)) \, dy
\]  

(19b)

After substituting (18) into (17), we obtain:

\[
C_{12}(z_1, z_2) = S^2 \cdot C_{xx}(u(x_0)) \cdot C_{yy}(v(y_0)) \cdot \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\omega_1 - \omega_2)H_z(\omega_1)H_z^*(\omega_2) \\
\times e^{i(\omega_1 z_1 - \omega_2 z_2)} \, d\omega_1 \, d\omega_2.
\]  

(20)

After evaluating the integral of (20) over \(\omega_1\) we have

\[
C_{12}(z_1, z_2) = S^2 \cdot C_{xx}(u(x_0)) \cdot C_{yy}(v(y_0)) \cdot \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} H_z(\omega_2)H_z^*(\omega_2)e^{i\omega_2(z_1 - z_2)} \, d\omega_2.
\]  

(21)

Notice that the integral term in (21) is the cross-correlation function between the axial system point response and its stretched version computed for an infinite observation window size. We label the normalized maximum of this function \(\rho_z\). Therefore, after axial tissue motion compensation (i.e., axial stretching of the pre-compression RF echo line), the maximum normalized correlation coefficient between the pre- and post-compression A-line of the tissue given by (5) and (7) is given by:

\[
\rho = \frac{C_{xx}(u(x_0)) \cdot C_{yy}(v(y_0))}{C_{xx}(0) \cdot C_{yy}(0)} \cdot \rho_z.
\]  

(22)

For a finite window size, \(\rho_z\) is given by:

\[
\rho_z = \max_{\tau} \left( \frac{1}{Z} \int_{-\frac{Z}{2}}^{\frac{Z}{2}} h_z(z)h_z^*(z - \tau) \, d\tau \right).
\]  

(23)

where \(Z\) is the observation window length and \(h_z^*(z)\) is the axial system impulse response after tissue motion compensation (stretched impulse response when the tissue is compressed). Thus far, the first two terms multiplying \(\rho_z\) have been assumed to equal unity, so that only the correlation coefficient \(\rho_z\) has been used to predict the elastogram quality \cite{10}. For that purpose, (23) was evaluated analytically for a given particular axial system impulse response \cite{11}, \cite{24}. For a better accuracy in predicting elastogram quality using the strain filter, (22) should be used instead. In this case the quality of an elastogram (SNR\_L) will depend on the spatial position of the ultrasound beam relative to the scanned target. In Section III experimental results corroborating these previous results are presented.

### III. Experimental Results

To verify experimentally the lateral degradation of elastograms due to tissue lateral motion the setup shown in Fig. 1 is used. The phantom is made of a mixture of gelatin, water, and small acoustic scatterers (graphite flakes) \cite{34}. To have a uniform axial strain distribution, the phantom is compressed using a rectangular plate in which the linear array transducer is inserted. Furthermore, we attempt to have perfect slip boundary conditions by lubricating both the upper and lower surface of the phantom with corn oil. The ultrasound system used for this experiment is a Disonics Spectra II real-time linear array scanner setup that operates with dynamic receive focusing and a single transmit focal zone centered at 30 mm. Center frequencies of 5 MHz and 7.5 MHz are used. As shown by Fig. 1, the digitized data were collected from a 40 x 40 mm ROI (starting at a depth of 5 mm under the transducer) centered around the transmit focus and for an applied strain ranging from 0.4 to 4% with steps of 0.2%. In each trial the ROI consisted of 100 A-lines. The overall elastography system is fully described in previous work \cite{13}.

Fig. 2 shows the correlation coefficient at the peak of the normalized cross-correlation function computed between pre- and stretched post-compression echo segments obtained at 5 MHz center frequency. In Fig. 2(a), these segments are taken from the central A-line (number 50) which corresponds to the scatterers that reside around the target axis of symmetry where, as discussed in the previous sections, the lateral motion is minimized. In Fig. 2(b) the segments are taken from the A-line close to the edge of the linear array transducer (number 2) where the lateral motion is larger. As expected, the peak of the normalized cross-correlation function decreases with increased applied strain in both cases. However, the correlation decreases faster in Fig. 2(b). This difference, as predicted by (22), is due to the increased decorrelation due to the motion of the scatterers across the ultrasound beam. Fortunately, the difference is not significant for relatively low tissue strain (< 1%). As will be shown later, when a higher center frequency is used (7.5 MHz), and thus the lateral resolution is improved, the difference becomes more significant even for low tissue strain.
Fig. 2. Maximum correlation coefficient versus applied strain. This correlation coefficient is measured using cross-correlation technique applied to data segment from pre- and uniformly stretched post-compression RF A-lines. The segment size is 3 mm. This correlation is an average of 10 overlapping segments. The error bars represent 2 standard deviations. (a) The correlation coefficients are measured using the center A-lines of the array. (b) The correlation coefficients are measured using the A-lines close to one edge of the target ($y_0 = 20$ mm).

In order to predict this difference using (22), prior knowledge of the beamwidth where the measurement of the correlation were taken is required. The beamwidth in the focus may be estimated from the experimentally measured average speckle size following the method proposed by Wagner et al. [29]. The same gel phantom described above is placed in a water tank and scanned for different lateral positions of the transducer; at each step the transducer is laterally moved by 0.15 mm. For each transducer position, an RF image of 100 A-lines is digitized. At the depth where the results of Fig. 2 were computed, the correlation coefficient is computed within a small observation window of $4 \times 4$ mm$^2$. Fig. 3 summarizes the results. Because the effective aperture size may change with lateral position, the beamwidth may also change. To verify this possibility, the correlation coefficients also are computed for three different positions relative to the scanned ROI: at the center, at mid-distance from the center, and near the edge. As shown by Fig. 3, in this particular case, the beamwidth remains nearly constant as we approach the edges of the linear array transducer.

If we assume that, at the focus the beam profile has a Gaussian shape, then the system point response in the lateral direction will be given by:

$$h_y(y) = (2\pi\sigma_y^2)^{-1/2} \exp(-y^2/2\sigma_y^2)$$

(24)

where $\sigma_y$, by definition, is a beam parameter (beamwidth $\approx$ FWHM $\approx 2.35\sqrt{2}\sigma_y$). The auto-covariance of such impulse response is given by [29]:

$$\frac{C_{yy}(\Delta y)}{C_{yy}(0)} = \exp(-\Delta y^2/4\sigma_y^2).$$

(25)

Therefore, the correlation coefficient in (22) for this particular case is given by:

$$\rho = \frac{C_{xx}(u(x_0))}{C_{xx}(0)} \cdot \exp(-v(y_0)^2/4\sigma_y^2) \cdot \rho_z.$$

(26)

Moreover, because the transducer is on the axis of symmetry of the target (phantom) in the elevational direction, the motion of the scatterers in that direction is close to zero and the first term in (26) is close to one. Hence (26) reduces to:

$$\rho \approx \exp(-v(y_0)^2/4\sigma_y^2) \cdot \rho_z.$$

(27)

As predicted by (27), for identical applied strain, a narrow beam (small $\sigma_y$) results in an increased decorrelation.

Using the definition of the full-width-at-half-maximum (FWHM) of a Gaussian function as an estimator of the beamwidth, the parameter $\sigma_y$ needed to predict the lateral decorrelation given by (27) is evaluated using the data
of Fig. 3 (FWHM ≈ 2.35√2σ_y). Following this, it is found that, near the edge of the array, σ_y ≈ 0.6 mm. Because the gel phantom is isotropic, and assuming perfect slip boundary conditions as presented by (8), for a given applied axial strain a the lateral displacement field is given by:

\[ v(y) = -\nu v_0, \quad -\frac{L_y}{2} \leq y \leq \frac{L_y}{2} \quad (28) \]

where \( \nu \) is the target Poisson's ratio. The phantom used here is incompressible as are most soft tissues. For such tissues a \( \nu = 0.5 \) is used in (28). Therefore, using both the experimentally measured parameter σ_y and (28), we can predict the lateral decorrelation for any applied strain and at any position (y) of the ultrasonic beam relative to the target. Fig. 4 compares the measured correlation of Fig. 2(b) to the one predicted using (27). From Fig. 4 we notice a good agreement, within an error bar, between the predicted and measured correlation.

The same set of experiments was repeated using a 7.5 MHz linear array transducer. Fig. 5 shows the lateral system autocorrelation function at two different positions; in the middle of the transducer and near one of its edges. As shown by Fig. 5, the measured beamwidth near the edge is wider than that in the middle of the linear array transducer. This difference is due to the use of a smaller effective array subaperture near the array edges. Using the same approach discussed previously, we found that, near the edge σ_y ≈ 0.36 mm, while in the middle σ_y ≈ 0.23 mm.

Fig. 6 shows the correlation peak as measured using the normalized cross-correlation function computed between pre- and stretched post-compression echo segments. In Fig. 6(a) these segments are taken from the central A-line (number 50) that corresponds to the scatterers residing around the target axis of symmetry where, as discussed in the previous sections, the lateral motion is the smallest. However, in Fig. 6(b) the segments are taken from the A-line close to the edge of the target (number 2) where the lateral motion is largest. As expected, due to the increased scatterer motion across the beam, the correlation decreases faster when measured near the edge. Using (27), the correlation coefficients measured near the edge are predicted from the correlation coefficients measured in the middle. As shown by Fig. 7, a good agreement between the predicted correlation and the measured correlation is found.

When comparing Fig. 4 to Fig. 7, it appears that the proposed model better predicts the lateral decorrelation for higher frequency. Such behavior may be related to our assumption of a constant lateral motion of the scatterers across the beam during tissue compression (12) that is valid unless the beam is relatively narrow and the applied strain is small. At 5 MHz we have measured a beamwidth of about 2 mm near the edge of the linear array transducer. Within such beam the tissue distortion may be significant and, therefore, the above assumption may be less valid and thereby, as shown by Fig. 4, results in an underestimation of the lateral decorrelation. On the other hand, at a frequency of 7.5 MHz, we measured a beamwidth near the edge of the array transducer of about 1 mm. Because, for such relatively narrow beam, the assumption of a locally constant lateral motion of the scatterers is better justified, our model better predicts the lateral decorrelation at this frequency compared to what is predicted at 5 MHz.

Furthermore, it is important to notice that, for the same applied strain, for the higher frequency (7.5 MHz) the difference between correlation measured near the target axis of symmetry and near the edges is larger compared to the one found when a lower frequency (5 MHz) is used (Fig. 3). This difference, as predicted by (22), is due to
the fact that the measured beamwidth at 5 MHz is almost twice that measured when a center frequency of 7.5 MHz is used. Therefore, there are two competing mechanisms at higher frequencies: the increase in $\rho_z$ as shown by Varghese and Ophiir [10] and at the same time the decrease of the beamwidth. One possible alternative is to decrease the applied strain, which in turn reduces the lateral scatterer motion. However, reducing the applied strain causes a reduction of the $SNR_e$. In Section IV we propose a method to avoid such tradeoffs.

IV. REDUCTION OF LATERAL NOISE IN ELASTOGRAMS

Obviously, the reduction of the lateral decorrelation noise in elastography could be achieved by reducing the lateral tissue motion. To do so, we propose to confine the tissue in the lateral direction. In this case the elevational tissue motion (out-of-plane motion) is doubled. Fortunately, in this direction the beam is wide and generally the acoustical scanning plane could be selected such as the elevational scatterer motion is small (i.e., a plane parallel and near the axis of symmetry of the elevational displacement field). To demonstrate this possible improvement, the following experiment was conducted. The same gel phantom used before was confined in the lateral direction between 2 rigid plates. It was then compressed up to 3.6% in steps of 0.2% each. At each step 100 A-lines were digitized. The same set of data was also collected when the gel phantom was not confined in the lateral direction. For this experiment the 7.5 MHz transducer was used. The transducer itself was used as the compressor. Figs. 8 and 9 summarize the obtained results.

Fig. 8 shows the correlation peak as measured using the normalized cross-correlation function computed between pre- and stretched post-compression echo segments as a function of applied strain. The two curves shown in Fig. 8(a) are obtained using the center A-line from the confined and unconfined target. Because the lateral motion is close to zero in the center (axis of symmetry of the lateral displacement field), both curves are essentially identical. However, as shown by Fig. 8(b), when the A-line close to the edge of the target is used, a significant increase of the correlation coefficient is measured when the target is confined. This increase is significant because, as predicted by the SF, even a small increase of the correlation coefficient results in a remarkable increase of the $SNR_e$. For example, for strains around 1% after confining the target, the correlation is increased from 0.8 to 0.9; using (1) the
is equivalent to a 7 dB increase of the sonographic SNR available to compute the tissue strain.

Fig. 9 shows two elastograms computed using one set of pre- and post-compression RF data collected after 2% compression, showing the unconfined [Fig. 9(a)] and the confined [Fig. 9(b)] phantom. The black pixels in these images correspond to an artificial zero strain level and is commonly called peak hopping noise. It is related to false peak detection of the lag of the maximum of the cross-correlation function between pre- and post-compression RF echo signals [31]. This noise occurs when the noises (electronic and decorrelation noise) increase the secondary correlation peaks above the primary correlation peak of the signal.
trations near the edges of the compressor (transducer). These stress concentrations are higher when the target is confined [Fig. 10(b)]. This difference is also shown by the elastograms of Fig. 9. However, due to the reduced lateral decorrelation, these strain concentrations are better depicted in the case of the confined target [Fig. 9(b)]. Away from the compressor edges, the tissue strains are the same. For our correlation analysis of the last section (Fig. 8) the RF echo segments are taken at midrange from the pre- and post-compression RF images where, as shown by Figs. 10 and 9(c), the tissue strain is equal for the confined and unconfined target.

V. Discussion

A complete understanding of the noise sources in elastography is the key for the improvement of image quality. As described earlier, these noise components consist of the sonographic noise (both electronic and quantization), and the decorrelation noise. In this paper, we focused our study on the decorrelation noise resulting from the motion of tissue scatterers relative to each other and relative to the transducer. More specifically, we studied the signal decorrelation due to the motion of the tissue scatterers in the lateral and elevational directions. We have shown that, for a uniformly elastic target, under the assumption that the system impulse response is separable (4), and that the components of the tissue displacement are 1-D functions (8) and constant within the beamwidth, the effective correlation measured using pre- and post-compression A-lines is the product of three terms accounting independently for tissue scatterers motion in the elevational, lateral, and axial direction (22). For a homogeneous medium, when the transducer and compressor are on the elevational axis of symmetry of the target, these three terms reduce to two: lateral and axial. It is shown that the lateral decorrelation depends on the beamwidth, the applied strain and the position of the beam in the target. This decorrelation is reduced near the target axis of symmetry where the lateral scatterer motion is the smallest. It is higher near the target edges where the lateral motion is the largest. The proposed theory was corroborated using a gel based uniform phantom.

As shown by Fig. 9(b), the lateral confinement of the gel phantom results in an elastogram with higher SNR and contains less peak hopping noise compared to the elastogram of the unconfined phantom [Fig. 9(a)]. Such improvement, also illustrated in Fig. 9(c) that shows a column near the right edge from each elastogram, is due to the reduction of the variance of the estimated displacement which results from the increase in the correlation coefficient between pre- and post-compression RF line [Fig. 8(b)].

Because the boundary conditions for the confined and unconfined configuration are different, it is expected that the resulting 3-D stress fields and the strain fields will be different. To study this difference, using a commercial finite element (FE) analysis software (Linear Stress, Algor, Pittsburgh, PA), we simulated both cases. Fig. 10 shows cross-sections from the resulting axial 3-D strain field from the unconfined [Fig. 10(a)] and confined [Fig. 10(b)] target. These cross-sections are taken at the same spatial location of the acoustical scanning plane used to generate the elastograms of Fig. 9. Notice the high strain concentrations near the edges of the compressor (transducer). These stress concentrations are higher when the target is confined [Fig. 10(b)]. This difference is also shown by the elastograms of Fig. 9. However, due to the reduced lateral decorrelation, these strain concentrations are better depicted in the case of the confined target [Fig. 9(b)]. Away from the compressor edges, the tissue strains are the same. For our correlation analysis of the last section (Fig. 8) the RF echo segments are taken at midrange from the pre- and post-compression RF images where, as shown by Figs. 10 and 9(c), the tissue strain is equal for the confined and unconfined target.
Fortunately, in this direction the beam is wide and the acoustical scanning plane of interest could be selected such as the elevational scatterer motion is small. The existence of such plane, however, will depend on both spatial organization of the tissue elastic inhomogeneities and the compressor size in the elevational direction. In practice, for breast elastography, this will require the design of a small (in the elevational direction) compressor (or use the transducer itself as a compressor) so that its edges in the elevational direction are far from the target (breast) edges. Such possible improvements of breast elastogram quality will be studied in future work.

We should add that the advantage of tissue confinement is to have a stationary elastographic noise. Indeed, without confinement, because the beamwidth and lateral tissue motion are dependent on spatial position, the effective correlation coefficient predicted by (22) will also be position dependent. Therefore, the elastographic performance will be position dependent. However, after confining the target, as shown by Figs. 9(a) and (b), the noise in the elastogram tends to be spatially uniform.

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REFERENCES

In Appendix we demonstrate the following relation

\[ E[T_1(\omega_1)T_2^*(\omega_2)] = S^2 \cdot C_{11}(u(x_0)) \cdot C_{22}(v(y_0)) \delta(\omega_1 - \omega_2). \]  
(A1)

Notice that for one A-line we have

\[ \int \int t(x, y, z)h_x(x)h_y(y) \, dx \, dy = [t(x, y, z) \otimes h_x(x)h_y(y)]_{x=0, y=0} \]  
(A2)

\[ \int \int t(x, y, z)h_x(x + u(x_0))h_y(y + v(y_0)) \, dx \, dy = [t(x, y, z) \otimes h_x(x)h_y(y)]_{x=u(x_0), y=v(y_0)}. \]  
(A3)

However, to demonstrate the relation in (A1), we consider many A-lines. Because the tissue scattering is modeled as a stationary process in 3 dimensions, when (A1) holds for many A-lines it is also valid for a single A-line. Using the Fourier transform relation the expectation term in equation A1 is written as:

\[ E[T_1(\omega_1)T_2^*(\omega_2)] = \int_{-\infty}^{\infty} E[t_1(z_1)t_2^*(z_2)] \times e^{i(\omega_2z_2 - \omega_1z_1)} \, dz_1 \, dz_2. \]  
(A4)

Now we write the expectation in (A4) for many A-lines:

\[ E[t_1t_2^*] = E \left\{ \int \int T(u, \nu, z_1)H_x(u)H_y(\nu) \times e^{i(xu + \nu \nu)} \, du \, d\nu \right\} \left\{ \int \int T(u, \nu, z_2)H_x(u)H_y(\nu) \times e^{i(xu + \nu \nu + \Delta xu + \Delta \nu \nu)} \, du \, d\nu \right\}^{*}. \]  
(A5)

In (A5), \( \Delta x = u(x_0) \) and \( \Delta y = v(y_0) \). After a simple change of variables we have

\[ E[t_1t_2^*] = \int \int \int \int E[T(u, \nu, z_1)T^*(u', \nu', z_2)]H_x(u)H_x^*(u') \times H_y(\nu)H_y^*(\nu')e^{i(xu + \nu \nu - xu' + \nu' \nu' - \Delta xu + \Delta \nu \nu)} \, du \, d\nu \, du' \, d\nu'. \]  
(A6)

Because the tissue scattering profile is modeled as a white Gaussian noise with a variance \( S^2 \), we have:

\[ E[T(u, \nu, z_1)T^*(u', \nu', z_2)] = S^2 \delta(u - u') \times \delta(\nu - \nu') \delta(z_1 - z_2). \]  
(A7)

Therefore, after substituting (A7) into (A6) and after evaluating the integration over \( u \) and \( \nu \) we found:

\[ E[t_1t_2^*] = S^2 \delta(z_1 - z_2) \int H_x(u')H_x^*(u')e^{i\Delta xu'} \, du' \times \int H_y(\nu')H_y^*(\nu')e^{i\Delta \nu \nu'} \, du'. \]  
(A8)