A Least-Squares Strain Estimator for Elastography

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A least-squares strain estimator (LSQSE) for elastography is proposed. It is shown that with such an estimator, the signal-to-noise ratio in an elastogram (SNR_e) is significantly improved. This improvement is illustrated theoretically using a modified strain filter and experimentally using a homogeneous gel phantom. It is demonstrated that the LSQSE results in an increase of the elastographic sensitivity (smallest strain that could be detected), thereby increasing the strain dynamic range. Using simulated data, it is shown that a tradeoff exists between the improvement in SNR_e and the reduction of strain contrast and spatial resolution.

KEY WORDS: Dynamic range; elastogram; elastography; imaging; least squares; noise reduction; sensitivity; strain; strain filter; ultrasound. © 1997 Dynamaedia, Inc.

1. INTRODUCTION

In living biological soft tissue, the differences in the molecular components and their microscopic and macroscopic structural organization results in differences in their stiffnesses. For example, in the normal breast, the glandular structure may be firmer than the surrounding fibrous connective tissue, which, in turn, is firmer than the subcutaneous adipose tissue. Moreover, the pathological state of these tissues is generally correlated with a modification of their corresponding stiffnesses. Indeed, many cancers, such as scirrhous carcinoma of the breast, appear as extremely hard nodules detectable by the standard medical practice of palpation. Although the practice of palpation is routinely used by physicians, it remains a subjective technique and is generally limited to the detection of only large, stiff tissue abnormalities located close to the skin surface.

In the past fifteen years, several efforts have been focused on deriving information related to tissue elastic properties. Most of these techniques are ultrasound based and have been reviewed by Ophir et al. In this paper, we are interested in one of these techniques that is named elastography and was first proposed by Ophir et al. This method is based on the estimation of tissue motion in response to the application of an external quasi-static small compression. The tissue motion is computed ultrasonically using a time domain cross-correlation technique applied to pre- and postcompression rf echo 1-D signals. To first order, under the assumption of a constant stress field, the axial tissue strains are used to represent a relative measure of tissue stiffness distribution. These local tissue strains, computed as the gradients of the measured displacements, are displayed as a gray scale image named elastogram.

The quality of the elastogram is mainly dependent on the precision of displacement estimates. Moreover, since the strain is computed using a derivative operation (Eq. (1)), the high frequency noise of the displacement data (small fluctuations in the displacement estimates) is further amplified. To illustrate this noise amplification, consider figure 1 that shows results obtained using a homogeneous gel phantom. This phantom is made of a mixture of gelatin, water, and small acoustic scatterers (graphite flakes).
Figure 1a shows a gray scale image of the axial displacement field measured within the scanned region of interest (ROI) in response to a 1% applied strain. Notice the high SNR of the measured displacement field. As shown by figures 1c and 1d, due to the amplification of the small fluctuations in the displacement estimates (Fig. 1b), the strain profile has a relatively poor SNR, resulting in a poor sensitivity that precludes the visualization of small, low contrast elastic tissue inhomogeneities. We define the displacement SNR as the ratio of the tissue displacement estimates to its standard deviation and the strain SNR or the elastogram SNR (SNR_e) as the ratio of the tissue strain to its standard deviation.

In order to improve the SNR in the strain image, O’Donnell et al. proposed to apply high strain to the tissue by accumulating many small compressions. In this case, assuming uncorrelated noise in the strain image, the strain variance is reduced by a factor of N, which is equivalent to increasing the SNR by a factor of $\sqrt{N}$ (where N is the number of accumulated...
single compressions). In practice, this would require a relatively long data acquisition time and therefore external random physiologic motion may add to the small applied motion.

In this paper, we propose to use a least-squares piecewise approach, rather than a gradient technique, in order to reduce the variance of the measured strain profile. Unlike the multicompression technique,\textsuperscript{5} our method requires less data and therefore less acquisition time. The strain filter, an analytical tool recently proposed by Varghese and Ophir\textsuperscript{6} is used to theoretically predict the increase in $SNR$, resulting from the use of this new strain estimator. The tradeoffs in term of spatial resolution and strain contrast are studied using computer simulations using a test object consisting of a background embedding a circular inclusion. Our measurement of these two parameters is qualitative and based on a measure of a global correlation between an ideal and estimated strain profiles.

The paper is organized as follows. In section 2, we present a short review of the steps behind the development of the SF used in elastography to predict the $SNR$ of an elastogram. In section 3, the principle of our method is presented and the expected improvement of the elastogram $SNR$ is illustrated using the SF. Experimental results are discussed in section 4. In section 5, tradeoffs in the increase of $SNR$, in term of reduction of spatial resolution and strain contrast are discussed with the help of a classical analytical solution of the elasticity equation combined with an ultrasound image formation model. Finally, in section 6, a summary and discussion of the contribution of the paper are presented.

2. BACKGROUND

Assuming a constant speed of sound, the axial tissue strain $s$, gradient of the displacement, is estimated from two adjacent time delay estimates separated by an interval $\Delta t$,\textsuperscript{3} viz.:

$$s = \frac{\tau_2 - \tau_1}{\Delta t},$$

(1)

where $\tau_1$ and $\tau_2$ are the time delays estimated for an observation window of duration $T$ at time $t$, and $t+\Delta t$ respectively. Therefore, the variance of the tissue strain is derived from the variance of the time delay or displacement estimates.

The variance of the estimated displacement in elastography has been investigated using a theory already established in different fields, such as in sonar and in radar, where time delay estimation is extensively used.\textsuperscript{7} This theory is based on the use of a theoretical lower bound known as the Cramér-Rao-Lower-Bound (CRLB) which is given by:

$$\sigma^2_{\tau,\text{CRLB}} = \frac{1}{2\pi^2 f_0^2 B T \text{SNR}_s \left(1 + B^2/12 f_0^2\right)},$$

(2)

where $T$ is the signal duration (observation time), $f_0$ is the center frequency, $B$ is the signal fractional bandwidth, and $\text{SNR}_s$ is defined as the ratio of the power in the signal to the noise power. This CRLB has been used by Céspedes et al.\textsuperscript{8} to study the performance of different interpolation techniques used in elastography to estimate subsample time delays of digitized echo signals.
Using Eq. (1), the CRLB bound of the tissue strain was derived by Cespedes et al. as

$$\sigma^2_{\epsilon,\text{CRLB}} = \frac{2\sigma^2_{\epsilon,\text{CRLB}}}{T\Delta t}.$$  \hspace{1cm} (3)

The observation window $T$ in Eq. (3) accounts for the covariance between two consecutive time shift estimates. As reported by Cespedes et al., since there is no analytical expression for the covariance as a function of the windows overlap, a linear combination of the contributions of the overlapping and nonoverlapping of the observation windows was used.\textsuperscript{7,10} When there is no overlap between the windows, then $T$ is substituted by $\Delta t$ in Eq. (3). In such a case, the two time shift estimates are uncorrelated.

The above CRLB bound applies only to pure time delay while in elastography we deal with an additional signal decorrelation due to physical tissue distortion. Recently, a modified version of the CRLB that includes signal decorrelation has been derived by Walker and Trahey.\textsuperscript{11} Such a bound is given by

$$\sigma^2_{\epsilon,\text{CRLB}} = \frac{3c}{4\pi^2 Z \left( B^3 + 12 B f_0^2 \right) \rho^2 \left( \frac{1}{SNR^2} + \frac{1}{SNR^2} \right)^2 - 1},$$ \hspace{1cm} (4)

where $\rho$ is the correlation coefficient between pre- and postcompression echo signals that accounts for signal decorrelation due to tissue distortion, $c$ is the speed of sound and $Z$ is the length of the correlation window size. Notice that in Eq. (4), the CRLB is expressed in mm using the relation $Z = cT/2$. In Eq. (4), the correlation coefficient $\rho$ is a function of the tissue strain, the observation window length and the imaging system parameters.\textsuperscript{12,13} Moreover, as shown by Weinstein and Weiss,\textsuperscript{14} the CRLB bound is applicable only for relatively high SNR. For lower SNR different lower bounds should be used. These are known as the Barankin bound, and the constant level bound that apply for moderate and low SNR, respectively. The combination of these lower bounds is referred as the Ziv-Zakai lower bound (ZZLB). Varghese and Ophir\textsuperscript{15} derived a modified ZZLB expression for the strain estimation variance ($\sigma^2_{\epsilon,\text{ZZLB}}$).

The elastographic signal-to-noise ratio (SNR) is defined as the ratio of the tissue strain to its standard deviation. Using this definition and the different expressions for the lower bounds of the strain variance, Varghese and Ophir\textsuperscript{5} have shown that, for a given set of imaging and signal processing parameters, when the SNR is plotted as a function of applied strain (equivalently tissue strain), the obtained curve has the shape of a bandpass filter. For this reason, such a variation of SNR with tissue strain was defined as the Strain Filter (SF).

The SF is a theoretical framework that is useful for predicting the quality of the strain estimates for a given set of imaging and signal processing parameters.\textsuperscript{15,16} However, this framework has thus far been evaluated for the gradient based strain estimator only. In this paper, we use an extended version of the SF to predict the quality of the elastogram computed using a least-squares strain estimator (LSQSE).

3. PRINCIPLES OF LEAST-SQUARES STRAIN ESTIMATOR

The main goal of the least-squares strain estimator (LSQSE) is the reduction of the noise amplification due to the gradient operation. This is achieved by a piecewise linear curve fit to the estimated displacement field. In this case, a 1-D displacement function $u$ sampled at M
points of the type shown in figure 1c, around a given small kernel of N points, may be modeled by

\[ u(i) = a z(i) + b, \quad \text{for} \quad i = 1, N \]  

(5)

where \( z \) is the tissue depth, and \( a \) and \( b \) are constants to be estimated. The constant \( a \) represents the local tissue strain. Equation (5) can be written in matrix format as

\[ y = A \begin{bmatrix} a \\ b \end{bmatrix} \]  

(6)

where \( A \) is an \( N \times 2 \) matrix for which the first row represents the depth position \( z(i) \) and the second is a row of ones. The classical least square solution of Eq. (6) is given by

\[ \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = A^T \left[ A A^T \right]^{-1} \hat{u}. \]  

(7)

where \( \hat{u} \) contains the noisy displacement data, and \( \hat{a}, \hat{b} \) are the LSQ estimates of \( a \) and \( b \). Notice that \( \hat{a} \) is an estimate of the tissue strain at the middle of the kernel. As shown below, the variance of this strain estimate is related to the variance of the displacements data and the number of points used. Notice that Eq. (7) could be modified to include a weighting matrix that accounts for the variance of the displacement estimates. A higher weight may be assigned to a low variance displacement estimate. The measured correlation coefficient could be used to construct such a weighting matrix.

The CRLB of the LSQ strain estimates, as in Eq. (3), is obtained from the CRLB of the linear regression fit problem. Such a CRLB is known from the literature and it is presented as follows:

\[ x[n] = b + an + w[n] \quad n = 0, 1, ..., N - 1, \]  

(8)

where \( w[n] \) is a white gaussian noise in the observation data set \( x[n] \), \( a \) is the slope and \( b \) is the intercept. Given the variance or the CRLB of the data noise (\( \sigma^2_{w,CRLB} \)), the CRLB of the variance of the estimate of \( a \) and \( b \) are given by

\[ \sigma^2_{b,CRLB} = \frac{2(2N-1)\sigma^2_{w,CRLB}}{N(N+1)}, \]  

(9a)

\[ \sigma^2_{a,CRLB} = \frac{12\sigma^2_{w,CRLB}}{N(N^2-1)}. \]  

(9b)

Assuming that the noise in the displacement data is a white gaussian random process with a CRLB bound \( \sigma^2_{w,CRLB} \) and using Eq. (9b), the CRLB of the strain estimates variance obtained using the LSQ method and assuming no overlapping between consecutive observation windows, is given by

\[ \sigma^2_{s,CRLB} = \frac{12\sigma^2_{u,CRLB}}{N(N^2-1)u^2}. \]  

(10)
FIG. 2 (a) A family of four strain filters plotted for the following fixed imaging system parameters: \( f = 5 \text{ MHz} \), \( B = 2.5 \text{ MHz} \), \( SNR = 40 \text{ dB} \), \( Z = cT/2 = 2 \text{ mm} \). \( \Delta x = c\Delta t/2 = 2 \text{ mm} \). The first SF from the bottom is obtained when the grid strain estimator is used (N = 2 in Eq. (10)). The other three SFs are obtained using the LSQSE for N = 3, 4 and 5, respectively. Observe the dramatic increase of the \( SNR \), resulting in a significant improvement of the sensitivity (smallest strain that could be seen in the elastogram), which in turn results in an increase of the strain dynamic range. (b) Plot of the \( SNR \) as a function of the number of displacement estimates used by the LSQSE for a 1% tissue strain computed using the same parameters as in (a). Observe the fast increase of the \( SNR \) as the number of points increases. The rate of such increase is the inverse of the expression given by Eq. (13).

The CRLB bound on the displacement estimates variance is an upper bound that is never reached in practice. For small applied strain, the displacement estimates variance approaches the CRLB. Notice that for the special case when N=2, the LSQ method is equivalent to the gradient technique and in this case the CRLB reduces to

\[
\sigma^2_{s,CRLB} = \frac{2\sigma^2_{u,CRLB}}{\Delta t^2}.
\]

Previous formulations of the SF (gradient method) have used Eq. (11) to predict the \( SNR \) for different tissue strain.\(^6\) For the LSQ method, we use the more general Eq. (10) to predict the \( SNR \) using the SF concept. Therefore, the bounds on the strain estimate variance are given by

\[
\sigma^2_{s,ZZLB} = \begin{cases} 
\frac{(s T)^2}{6 \Delta t^2}, & \gamma < BT SNR_C < \delta \\
12\sigma^2_{\tau,BB}, & \delta < BT SNR_C < \mu \\
\Delta t^2 N (N^2 - 1), & \mu < BT SNR_C < \eta \\
12\sigma^2_{\tau,CRLB}, & \eta < BT SNR_C
\end{cases}
\]

In figure 2a, a family of four strain filters is plotted for the following fixed imaging parameters: \( f = 5 \text{ MHz} \), \( B = 2.5 \text{ MHz} \), \( SNR = 40 \text{ dB} \), \( Z = cT/2 = 2 \text{ mm} \). \( \Delta x = c\Delta t/2 = 2 \text{ mm} \). The first SF from the bottom is obtained when the gradient strain estimator is used (N = 2 in Eq.
The other three SFs are obtained using the LSQSE for $N = 3, 4$ and $5$, respectively. Since, as shown by Eq. (12), the CRLB of the strain variance is inversely proportional to approximately $N^3$, for $N \geq 3$, a dramatic improvement of the SNR is obtained. Such an increase in SNR results in a significant improvement of the elastographic sensitivity (left hand side of the SF; smallest strain that could be detected in the elastogram), thereby increasing the tissue strain dynamic range (DR) (width of the SF at a given SNR level). Notice that even the use of only three displacement estimates to compute the local tissue strain results in doubling the SNR. Figure 2b shows the SNR at 1% tissue strain as a function of the number of data points used by the LSQSE. Observe the rapid increase of the SNR as the number of data points increases. Such an increase in SNR is due to an equivalent decrease of the strain estimates standard deviation (approximately as $N^{-3/2}$). Using Eq. (12), the rate of such a decrease, $(r)$, in the CRLB region, computed as the ratio of the standard deviation of the strain obtained using the LSQSE to the standard deviation of the strain obtained using the gradient technique, is given by

$$r = \frac{6}{\sqrt{N(N^2 - 1)}} \approx \frac{\sqrt{6}}{N^{3/2}}.$$ (13)

4. EXPERIMENTAL RESULTS

The same elastogram shown in figure 1b is computed using the LSQSE applied to the displacement field shown in figure 1a. The resulting elastogram is shown in figure 3a. In this case, a kernel of seven displacement estimates is used. Notice the dramatically reduced variance of the elastogram of figure 3a. This is illustrated by figure 3b, which compares the middle row from the elastogram of figure 3a to the corresponding row from the elastogram of figure 1b.
FIG. 4 Reduction of the standard deviation of the strain estimates as a function of the number of displacement estimates used to compute the local strain. (—): Measured average rate of the decrease of the standard deviation of the strain estimates with the number of data points used by the LSQSE. The error bars represents two standards deviations. (---): Equivalent predicted rate given by Eq. (13).

As shown by figure 4, and as predicted by the SF, the standard deviation of the estimated strain decreases as the number of points used by the LSQSE increases. Observe that the rate of this decay follows the same rate predicted by Eq. (13).

5. TRADEOFFS OF LSQSE

As shown before, as the number of displacement estimates used to compute the local tissue strain increases, as the SNR increases (Fig 2b). In practice, for an inhomogeneous elastic medium, there is a tradeoff between spatial resolution, the strain contrast and the increase in SNR. In this section, we use the classical analytic solution obtained for an infinite medium perfectly embedding a bounded circular elastic inclusion combined with an image formation model to study such tradeoffs. For more details on the analytic solution of the elasticity equations and the image formation model, the reader is referred to previous papers.18,19

Figure 5a shows the stiffness distribution in an ROI from an infinite medium embedding a circular bounded inclusion. The size of the ROI is 40 x 40 mm; the inclusion is 5 mm in diameter and is three times stiffer than the background. Figure 5b shows the strain image (ideal elastogram), in the same ROI, resulting from a remote uniform uniaxial compression of the infinite medium assumed to be in a plane-strain state.18,19 As shown by figure 5b, the elastogram illustrates the inclusion with a shadowing inherent to the high stress concentration at the inclusion boundaries.

Using the image formation model discussed by Kallel and Bertrand,19 pre- and post-compression rf ultrasound images from the ROI of figure 6a are generated. For this simulation, we considered a linear array transducer with a center frequency of 5 MHz, 50% fractional bandwidth, 1 mm beamwidth, a fully developed speckle tissue acoustic scattering function with a mean of 13 scatterers/wavelength, a 48 MHz sampling frequency in the axial direction and 0.4 mm/pixel in the lateral direction. Such parameters are selected to represent our Diasonics Spectra II ultrasound scanner when it operates at 5 MHz in the focal zone.
Using the crosscorrelation technique applied to the simulated pre- and postcompression rf echo signals, the simulated tissue displacement is computed for a given 2 mm widow size (Z = 2 mm) and 1.5 mm overlap (Az = 0.5 mm). Using a different number of displacement estimates, the tissue strain image is then computed using the LSQSE. Figure 6a shows the ideal elastogram compared to the elastogram computed from the ultrasound data using the LSQSE for different number of displacement estimates (noisy elastogram). From figure 6b, we notice that the SNR increases with the number of displacement estimates used to compute the tissue strain, thereby increasing the elastographic sensitivity defined as the smallest tissue strain that could be depicted from the elastogram background noise. This increase in sensitivity is manifested here by the dramatically improved visualization of the dark (hard) "X" artifact centered at the center of the lesion. This result is predicted by the strain filter (Fig. 2b).

Figure 7a shows the rate of the decay of standard deviation of the LSQ strain estimates, computed in the background, as a function of the number of points used by the LSQSE. The corresponding SNR is shown in figure 7b. Notice the rapid decrease of the standard deviation of the strain estimates with the increase of the number of points used by the LSQSE. As shown by figure 7a, the rate of decay of the strain standard deviation predicted by Eq. (13) follows the measured one obtained from simulated data. Equivalently, as shown by figure 7b, the reduction of the strain estimate variance results in an important increase of the SNR. Unfortunately, in practice, where generally we deal with inhomogeneous medium, such a significant increase of SNR is reached at the expense of a reduction of spatial resolution and strain contrast. Therefore, a tradeoff exists between the increase of SNR and the equivalent reduction of contrast and spatial resolution. From figure 6b, we notice that in this case, the elastogram obtained using eight points gives the best tradeoff between SNR, contrast and spatial resolution since it gives the best overall correlation with the ideal strain image (Fig. 6a). To confirm quantitatively such results, the correlation coefficient between 10 columns around the middle row from the noisy elastogram and the corresponding 10 columns from the ideal elastogram is computed. The middle column from the ideal strain image is plotted in figure 8a with the corresponding columns from the noisy elastograms obtained using the LSQSE with 2, 8 and 17 data points. Figure 8b shows the correlation coefficient between the
ideal strain and the noisy elastogram as a function of the number of points used by the LSQSE.

As shown by figure 8b, the correlation rapidly increases with the number of points used by the LSQSE to reach a maximum when eight points are used and then slowly decays as the number of points increases. In this case, for the particular window size and window overlap used to compute the tissue displacement, eight points corresponds to 4 mm, which is close to the size of the inclusion (5 mm). As shown by figure 8a, when the number of points corresponds to a size equal to or greater than the width of the inclusion, the high frequency components (rapid strain transitions) of the strain profile are highly attenuated, which is equivalent to a low pass filtering effect. Therefore, in general, the optimal number of points will depend
FIG. 7 Decrease and increase, respectively, of the strain standard deviation and the SNR. (a) (—): Measured average rate of the decreasing of the strain estimates standard deviation with the number of data points used by the LSQSE. The error bars represent two standard deviations. (---): Equivalent predicted rate using Eq. (13). (b) SNR of the strain estimates versus the number of data points used by the LSQSE. The standard deviation and the SNR are computed in the background and averaged over 10 uncorrelated estimates computed from different locations in the elastogram.

FIG. 8 Tradeoff in the increase of SNR in terms of the reduction of strain contrast and spatial resolution. (a) Columns from the ideal elastogram and the elastograms obtained using 2, 8 and 17 data points. (b) Correlation coefficient between the middle row from the ideal elastogram and the noisy elastogram as a function of the number of data points used by the LSQSE. The correlation coefficient is averaged over the 10 columns around the middle of the image.

on the spatial frequency components of the strain profile. When the strain varies slowly, a large number of points may be used to have a higher SNR.

In order to compare the tradeoffs of the LSQSE to those of a low pass filter used by O'Donnell et al to reduce the noise of the strain image, we reproduced the same results shown by figure 7b and 8a using the kernel size of the moving average filter. Figure 9a shows the increase of the SNR as a function of the kernel size of the moving average filter. From
In this paper, a least square strain estimator (LSQSE) is proposed to reduce the noise amplification due to the gradient operation in the current technique. Using the strain filter (SF), we have shown that the LSQSE results in a significant increase in $SNR_c$ thus improving the sensitivity, which in turn results in an increase of the strain dynamic range. Results obtained from a gel phantom and simulation are used to illustrate such improvement in elastogram quality.

It is shown that with the LSQSE, the $SNR_c$ increases as a function of the kernel size (number of displacement estimates used to compute the strain). A closed-form expression predicting the rate of such increase in $SNR_c$ is obtained using the SF. Both simulation and experimental results are used to corroborate this expression.

In practice, where we deal with inhomogeneous tissues, the increase in $SNR_c$ is achieved at the expense of a reduction of strain contrast and spatial resolution. Therefore, a tradeoff
FIG. 10 Graphical comparison of the moving average filter and the LSQSE. (a) Elastogram generated using a moving average filter applied on the elastogram obtained using the gradient technique. (b) Elastogram obtained using the LSQSE. (c) Ideal elastogram. The elastograms in (a) and (b) are both obtained using the same kernel size (9 data points). The elastograms are displayed using the same gray scale dynamic range. Observe that the elastogram in (b) shows the inclusion with a better contrast compared to the elastogram in (a).

exists between the increase of the kernel size and reduction of spatial resolution and contrast. Using a classical analytic solution of the elasticity equation combined with an image formation model, these tradeoffs are analyzed. It is shown that the optimal kernel size depends on the tissue strain distribution and, more specifically, on its high frequency components (rapid transitions). When the tissue strain profile is smooth, which is equivalent to having a smooth continuous stiffness distribution, a high number of data points may be used by the LSQSE in order to have a high SNR. However, when the strain profile contains high spatial frequencies a smaller kernel should be used. Therefore, in practice, we suggest the use of variable kernel sizes to generate the elastogram. The local optimal kernel size will be dependent on the spatial gradient of the strain variation.

Finally, using simulated data, we have shown that the LSQSE is superior to a moving average filter for reduction of the elastogram noise and for increasing elastographic sensitivity.

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REFERENCES


